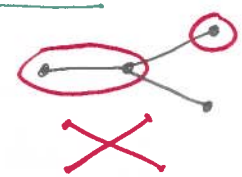
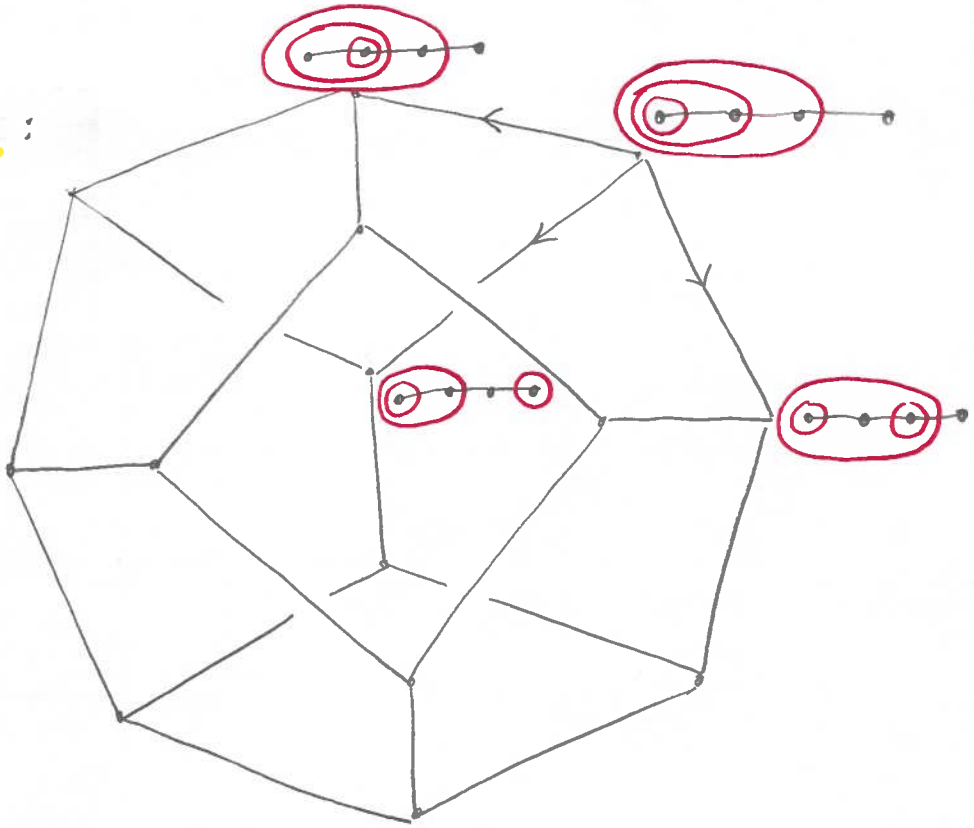
Definitions: **Tube** t on a graph:
induced, connected, subgraph.

Tubing T: a set of tubes,
each pair nested or disjoint,
and unions of them must be induced subgraphs

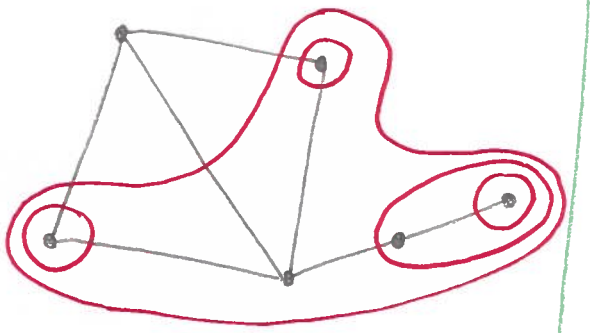


Path:



Associahedra
[Stasheff]

Any graph:



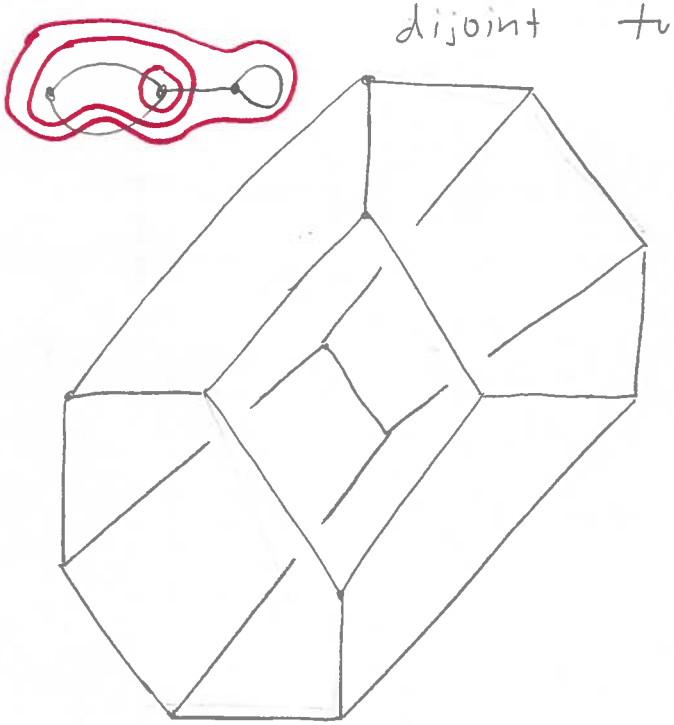
Graph - Associahedra.
[Carr, Devadoss]

Definitions: **Tube** t on a pseudograph:

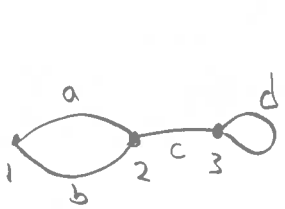
Filled, connected, sub-pseudograph.

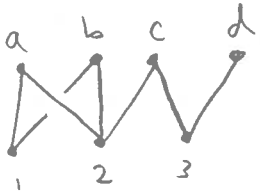
↳ [if end-nodes of a (multi) edge(s) are in t then at least one of those multiedges is in t]

Tubing T: a set of pairwise nested or disjoint tubes with filled unions.



Notice:





pseudograph ↔ poset (Hasse diagram)

Pseudograph Associahedra

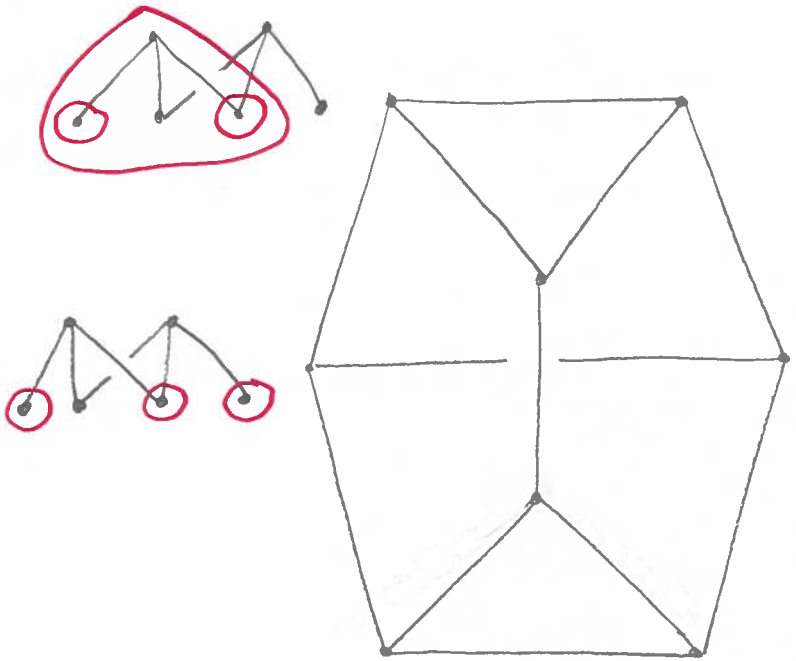
[Carr, Devadoss, F.]

Definitions: **Tube** t on a poset:

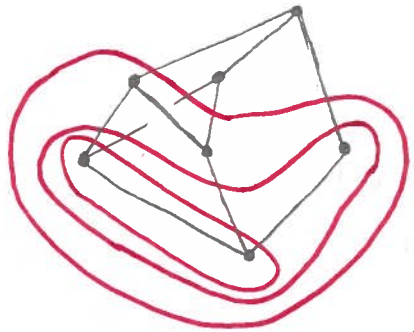
filled, connected, lower set.

↳ [if t contains the set ∂x of all $y < x$ then t intersects the set b_x of y s.t. $\partial y = \partial x$.]

Tubing T: a set of pairwise nested or disjoint tubes with filled unions.



Poset Associahedra



[Devadoss, F., Reisdorf, Showers]

Questions:

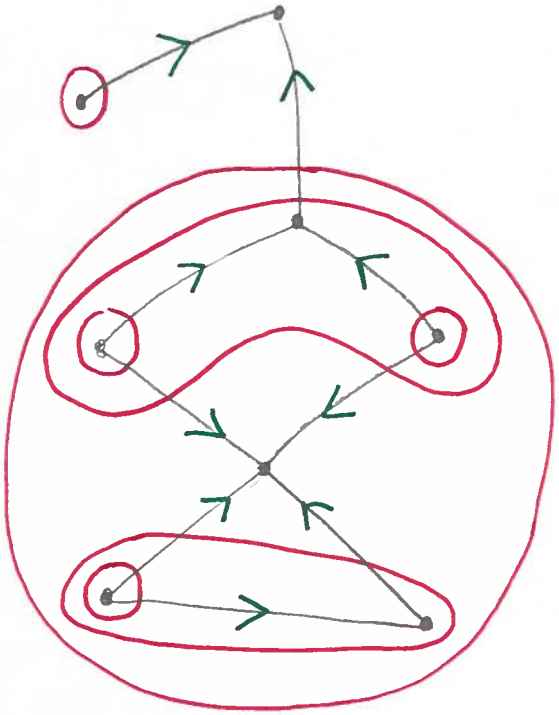
- Find a realization of the poset associahedra. [Katz, Olsen] ↳
- Find formulas for h-vectors, $h(t,q)$.
- Relate to Galashin's poset associahedra.

Tubings $\longrightarrow$ Orientations

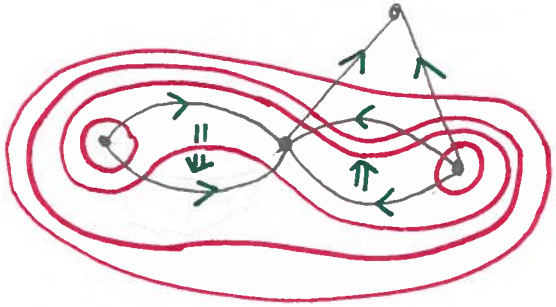
$T \longmapsto O_T$

where $O_T(H)$ chooses the element $z \in H$ such that $z \in t \Rightarrow H \subseteq t$.

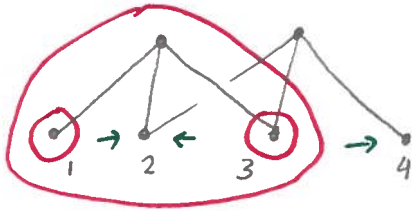
[H is any $\partial_c y \subseteq b_x$, or any non-minimal b_{xy}, and $\partial_c y$ = maximal elements of ∂_y]

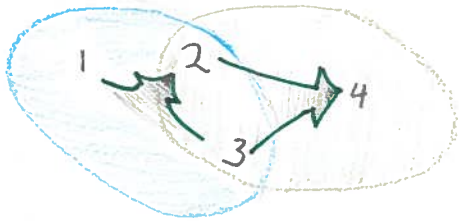


- Directed Acyclic Graph



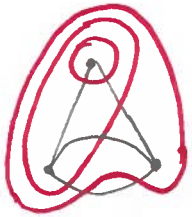
- Directed Acyclic Pseudo graph
- Pasting diagram

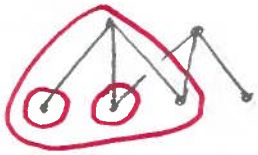


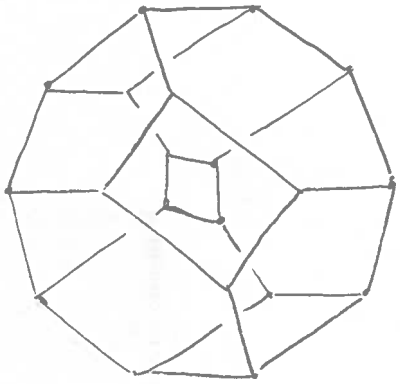


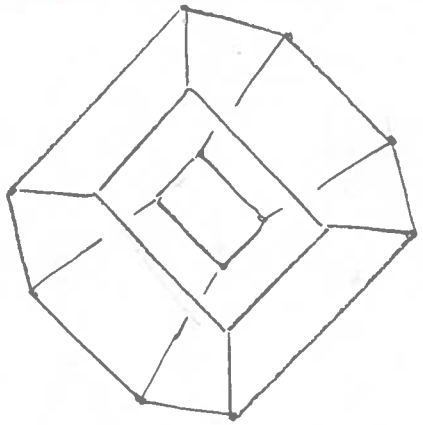
- Directed Acyclic Hypergraph
- Poset Gradient

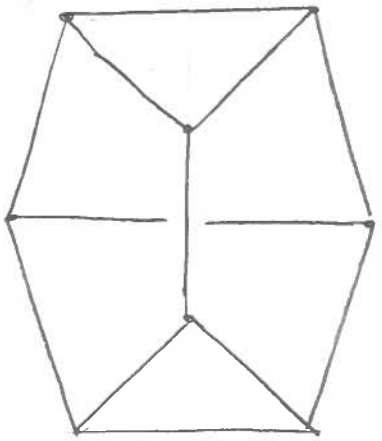
Polytope fan refinements



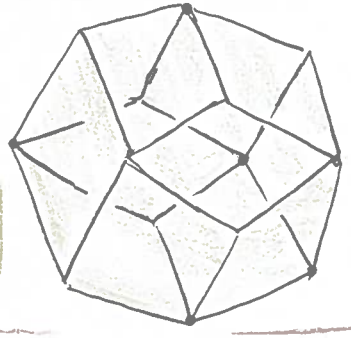






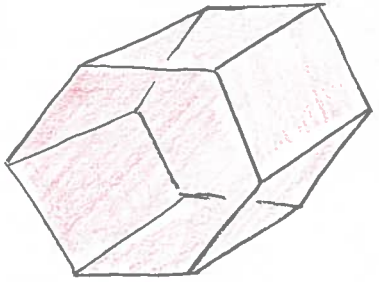


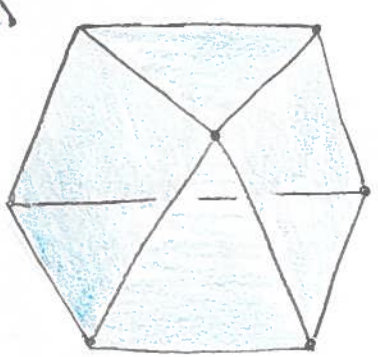
↓ Open Question!



graphical zonotope

[Féray, Reiner]





Hypergraphic polytopes

[Benedetti, Bergeron, Machacek]

[Bergeron, Pilaud]

Q.
- Are all poset gradient polytopes products of hypergraphic polytopes?