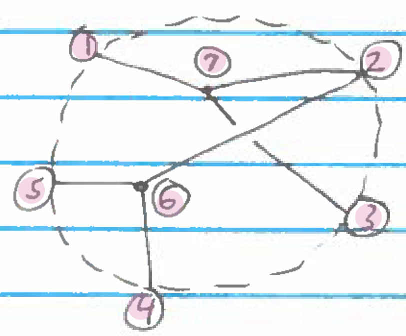
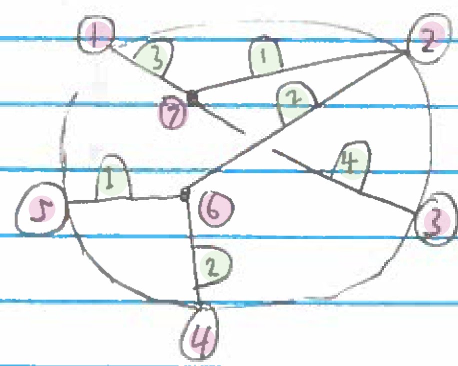
The experiments.

1) Make up a non-planar network



2) Assign positive numbers to each wire: conductances



3) Make a matrix called L.

diagonal entry L_{ii} = negative sum of wires touching i

$$
\begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
-3 & 0 & 0 & 0 & 0 & 0 & 3 \\
0 & -3 & 0 & 0 & 0 & 2 & 1 \\
0 & 0 & -4 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & -2 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 2 & 0 & 2 & 1 & -5 & 0 \\
3 & 1 & 4 & 0 & 0 & 0 & -8
\end{bmatrix}
\end{array}
$$

off-diagonal entry L_{ij} = conductance of wire i to j (0 if no direct connection)

4) Break up L into chunks:

$$\begin{array}{c|c|c} & \text{exterior nodes} & \text{interior nodes} \\ \hline \text{exterior nodes} & A & B \\ \hline \text{interior nodes} & B^T & C \end{array}$$

$$A = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -3 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c c} & \begin{array}{cc} 6 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 3 \\ 2 & 1 \\ 0 & 4 \\ 2 & 0 \\ 1 & 0 \end{bmatrix} \end{array}$$

```
>> B=[0 3; 2 1; 0 4; 2 0; 1 0]
```

```
>> A=[ -3 0 0 0 0; 0 -3 0 0 0; 0 0 -4 0 0; 0 0 0 -2 0; 0 0 0 0 -1]
```

$$C = \begin{array}{c c} & \begin{array}{cc} 6 & 7 \end{array} \\ \begin{array}{c} 6 \\ 7 \end{array} & \begin{bmatrix} -5 & 0 \\ 0 & -8 \end{bmatrix} \end{array}$$

```
>> C=[-5 0; 0 -8]
```

5) Find the response conductance matrix

$$M = A - BC^{-1}B^T$$

```
>> format rat
>> M=A-B*(C^(-1))*B.'
```

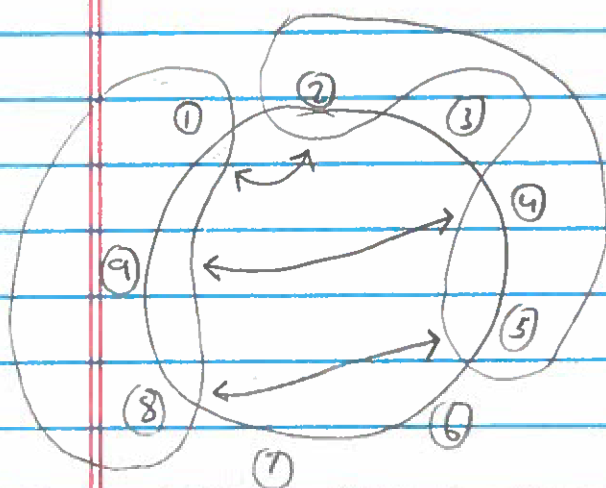
M =

$$\begin{bmatrix} -15/8 & 3/8 & 3/2 & 0 & 0 \\ 3/8 & -83/40 & 1/2 & 4/5 & 2/5 \\ 3/2 & 1/2 & -2 & 0 & 0 \\ 0 & 4/5 & 0 & -6/5 & 2/5 \\ 0 & 2/5 & 0 & 2/5 & -4/5 \end{bmatrix}$$

6) Check that some circular minors of M are negative.

Circular minor:

(a) choose any two sets of the exterior nodes, both in order on the circle, non-interlaced (I can circle them separately!), both with same number k of nodes



ex: $\boxed{k=3}$

(b) Pick either set, and use its numbers to choose k rows of M (Easiest: pick the one that doesn't wrap around the circle back to 1)

R = rows: $\{2, 4, 5\} = R$, in increasing order.

(c) Across from each node in R, in the circle, is a node in the other set G, standing for a column of M. Choose those columns, in the order from their matching with R; $G = \{1, 9, 8\}$ to make a smaller matrix $M(R,G)$

$1 \longleftrightarrow 2$

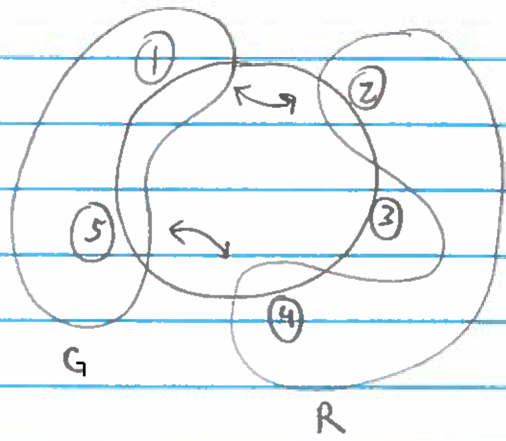
$9 \longleftrightarrow 4$

$8 \longleftrightarrow 5$

(d) Find $(-1)^k \det M(R,G)$.

For our M:

$$\begin{bmatrix} -15/8 & 3/8 & 3/2 & 0 & 0 \\ 3/8 & -83/40 & 1/2 & 4/5 & 2/5 \\ 3/2 & 1/2 & -2 & 0 & 0 \\ 0 & 4/5 & 0 & -6/5 & 2/5 \\ 0 & 2/5 & 0 & 2/5 & -4/5 \end{bmatrix}$$

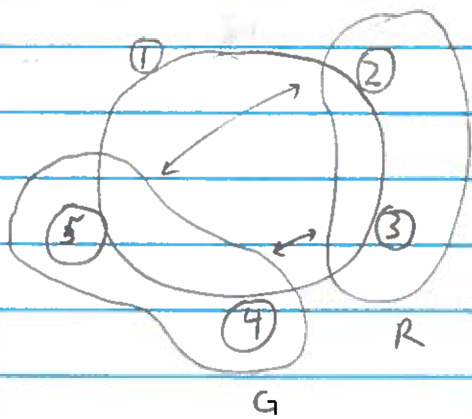


$k=2$

$(-1)^2 \det M(R,G)$

$= 1 \cdot \det \begin{bmatrix} 3/8 & 2/5 \\ 0 & 2/5 \end{bmatrix}$

$= 6/40 \geq 0$



$k=2$

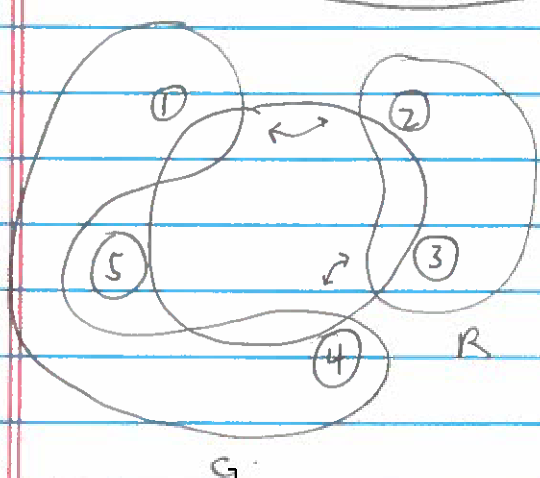
$(-1)^2 \det M(R,G)$

$$\begin{bmatrix} -15/8 & 3/8 & 3/2 & 0 & 0 \\ 3/8 & -83/40 & 1/2 & 4/5 & 2/5 \\ 3/2 & 1/2 & -2 & 0 & 0 \\ 0 & 4/5 & 0 & -6/5 & 2/5 \\ 0 & 2/5 & 0 & 2/5 & -4/5 \end{bmatrix}$$

$= 1 \cdot \det \begin{bmatrix} 2/5 & 4/5 \\ 0 & 0 \end{bmatrix}$

Note! These columns are in reverse order since they are reordered according to the matching (directly across the circle) with the ordered rows.

$= 0 \geq 0$



$k=2$

$$\begin{bmatrix} -15/8 & 3/8 & 3/2 & 0 & 0 \\ 3/8 & -83/40 & 1/2 & 4/5 & 2/5 \\ 3/2 & 1/2 & -2 & 0 & 0 \\ 0 & 4/5 & 0 & -6/5 & 2/5 \\ 0 & 2/5 & 0 & 2/5 & -4/5 \end{bmatrix}$$

$(-1)^2 \det M(R,G)$

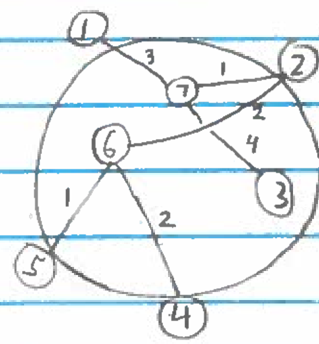
$= 1 \cdot \det \begin{bmatrix} 3/8 & 4/5 \\ 3/2 & 0 \end{bmatrix}$

$= -12/10 < 0$

7) Next: For each four nodes, let the other remaining nodes be pushed inside the circle.

Then repeat steps 1) through 6), with the exterior nodes just the four and interior now including the others.
To make L, A, B, C, ; list the pushed nodes very last.

ex: push node 3
(keep 1 2 4 5)



New L =

$$
\begin{array}{c c}
 & \begin{array}{cccc|ccc} 1 & 2 & 4 & 5 & 6 & 7 & 3 \end{array} \\
\begin{array}{c} A \\ \\ \\ \\ B^T \\ \\ \\ \end{array}
\begin{array}{c} 1 \\ 2 \\ 4 \\ 5 \\ 6 \\ 7 \\ 3 \end{array} &
\left[\begin{array}{cccc|ccc}
-3 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & -3 & 0 & 0 & 2 & 1 & 0 \\
0 & 0 & -2 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 \\
\hline
0 & 2 & 2 & 1 & -5 & 0 & 0 \\
3 & 1 & 0 & 0 & 0 & -8 & 4 \\
0 & 0 & 0 & 0 & 0 & 4 & -4
\end{array}\right]
\begin{array}{c} \\ B \\ \\ \\ \\ C \\ \\ \end{array}
\end{array}
$$

Find new $M = A - B C^{-1} B^T$

Test circular minors: We expect that there will be four nodes from original that give negatives!