

Project Summary:
Finite type invariants and grope constraints on braids and string links.

For knots K_1 and K_2 the following are equivalent:

- (i) K_1 and K_2 have the same finite type invariants of Vassiliev degree $< n$.
- (ii) K_1 and K_2 are cobordant by a capped grope of class n .
- (iii) $K_1 =$ the closure of a braid b and $K_2 =$ the closure of pb , where $p \in LCS_n(P_k)$.

Several questions arise. First, points (i) and (iii) are still equivalent with finite type invariants replaced by delta finite type invariants and the pure braids P_k replaced by the commutator subgroup of the pure braids. It is an open question whether there is a grope cobordism formulation of delta finite type invariants. Secondly, the statements above lead to an equivalence relation on knots first formulated by Goussarov and Habiro. These equivalence classes form abelian groups under connected sum for knots but not for links. It is an open conjecture that string links have the same behavior under composition with elements in the lower central series of pure braids as do knots. They do form a nonabelian group under the Vassiliev equivalence. Finally, a pure braid bounds the image of a grope of class n mapped continuously into $\text{Config}(D^2, k)$. It is an open question whether there is any direct relation between this grope and that of point (ii) above.

The principal investigator aims to shed light on the preceding questions partly through the study of a new and highly structured family of filtrations of the braid groups that will be referred to as grope constrained subgroups. A grope of type T is designated for each pair of parentheses in a partition of the strands of B_n , and the constrained braids are those in which each subset of strands is contained in an annular grope of the given type. The filtration is tightened either by passing along face maps in the associahedron or by passing to gropes of greater class. The plan is to elucidate relationships between the filtrations given by grope constraints and the lower central series and derived series of the pure braids. It would especially be of value to relate the grope constrained groups to the special subgroups in the descending series: Brunnian braids and the more general k -trivial braids. It is expected that the constraining gropes will be comparable to the class n gropes in the equivalent statements above, enabling a search for canonical cobordisms and continuous maps.

Symmetric grope cobordism projects onto the Cochran–Orr–Teichner filtration of the knot concordance group. Since monotonicity is not required of string links then gropes of higher complexity and symmetry contribute to the structure of grope constrained submonoids of string links than for braids of similar size. It seems wise to look for a connection between symmetric grope cobordism, constrained string links, and knot equivalence modulo the derived series of the pure braids. It is a related goal of the principal investigator to formulate topological quantum field theory versions of finite type invariants. If concordance invariants can be constructed, perhaps as a limit of symmetric grope cobordism invariants, then the open questions of torsion in the concordance group could be attacked. Professor Forcey also plans to look for relations between the quotients of grope constrained subgroups of braids over S^n and the homotopy groups of S^{n+1} .

The research into low dimensional topology discussed here will be performed largely at Tennessee State University, a historically black university (HBCU) with a large proportion of minority students. As a faculty member there Professor Forcey will advise undergraduate senior mathematics majors and masters degree candidates on their required theses. This project will be a valuable source of questions for student research, and will also encourage networking with the counterparts of TSU in the region and wider academic community. In addition, TSU is working towards the establishment of a Doctoral program in mathematics which when in place will be only the second such program at an HBCU in the U.S.. Steps in that direction include the hiring of new faculty actively involved in research, and the procurement of research grants to help continue that activity.