

The formulas:

$W(M)$: Given M an $n \times n$ response matrix
for a connected N (n exterior nodes)

$$W = DJ + JD - 2(-M)^{\dagger}$$

where $J = n \times n$ all 1's

$D =$ diagonal entries of $(-M)^{\dagger}$

$M(W)$: Given W an $n \times n$ resistance matrix
(non-zero off diagonal)

$$M = \left(\frac{1}{2} (W - \frac{1}{n} (WJ + JW)) + \frac{1}{n^2} \text{trace}(WJ)J \right)^{\dagger}$$

The onto problem for $n=4$:

Given

symmetric $W =$
$$\begin{bmatrix} 0 & a+b+f & a+c+e+f & a+d+e \\ \text{non zero off} & & 0 & b+e+c & b+d+e+f \\ \text{the diagonal,} & & & 0 & c+f+d \\ & & & & 0 \end{bmatrix}$$

with $ac \geq ef$

$bd \geq ef$

show $M(W)$ is a response matrix
for a circular planar N .

(use symbolic Matlab)

The range problem for $n \geq 5$

Find the range $\{W(M) \mid M \in \tilde{EP}_n\}$

that is, M is the response matrix for a circular planar connected network N , with n exterior nodes.

Question: has anyone studied the space of resistance matrices, or resistance metrics?

Special cases might include: all resistance/conductances are value 1.

Planar graphs? Outer-planar graphs?