Spring 2020 Intro to Topology 3450:445-545

INSTRUCTOR: Dr. Stefan Forcey EMAIL: sforcey@uakron.edu
OFFICE: CAS 275 PHONE: 972-6779
OFFICE HOURS: MWF 2:45-3:45pm, and lots more by appointment!

TEXTs:

[PS] Hatcher, Point-Set Topology:
http://www.math.cornell.edu/~hatcher/Top/Topdownloads.html

[AT] Hatcher, Algebraic Topology:
http://www.math.cornell.edu/~hatcher/AT/ATpage.html

BIBLIOGRAPHY:

Prasolov, V.V., *Intuitive Topology* A.M.S.
Weeks, Jeffrey. *The Shape of Space*.

Website for schedule, homework problems and announcements:
http://www.math.uakron.edu/~sf34/class_home/topo/topos20.htm

GRADING POLICY:

400 pts: Homework
200 pts: Present Homework Solutions
400 pts: Project.

900 pts. guarantees an A
800 pts. guarantees a B
700 pts. guarantees a C
600 pts. guarantees a D
(+,- at my discretion)

Course Outline:

Jan. 13 Category of topological spaces and continuous maps.

Reading [PS] Chapter 1,2 pgs 1-20.

1) ______________________Given the spaces

X={1,2,3,4,5,6,7}; $\mathcal{O}_X$={{},X,{1},{2},{1,2},{2,3},{1,2,3},{1,2,3,7}} and

Y={y in **N** | 0 < y < 15};

$\mathcal{O}_Y$ = {{},Y,{2},{5},{2,5},{2,4,5},{1,2,5,7},{1,2,4,5,7},{1,2,4,5,6,7},{1,2,4,5,6,7,10}};

and Z={2,4,6,8}; $\mathcal{O}_Z$={{},Z,{2}{4,6}{6,8}}.

Which pair A, $\mathcal{O}_A$ is not a topology? Find bases for the two topological spaces. Is the function $f = 2x$ continuous from X to Y? Is the function g from the unit interval given by $g(t) = 2$ for $t < ½$, and $g(t)$=4 otherwise, a path in X? Is that same function a path in Y?

Pick 2 from weeks 2-7, and another 2 topics from weeks 8-12. Email me!

Jan. 21 Reading [PS] Chapter 2 pgs 20-28.

2) ________________________Connectivity, path connectivity.

Is the space X, $\mathcal{O}_X$ from the previous week path connected? Is it connected? Describe a specific space that is connected but not path connected. Describe a specific space that is connected but for which it is unknown whether it is path connected. For what topologies on the set of vertices of a graph are the edges of the graph connected as subspaces?

Feb 3 Metric spaces. Reading [PS] pgs 9-10. Also see the links below.

3) _____________Draw the unit balls in the taxicab metric (see the wiki: https://en.wikipedia.org/wiki/Taxicab_geometry) and the bus metric (also known as the British Rail metric, on page 94 here: https://www.math.ucdavis.edu/~hunter/m125a/intro_analysis_ch7.pdf). Show that a continuous function from Calculus 1 is a continuous function in the standard topology of the real line.

Feb 10 Gluing and cutting spaces: Quotient Spaces: Reading [PS] pgs 20-21

4) _____________ENUMERATE THE CUT POINTS, NON-CUT-POINTS, AND THE HOMEOMORPHISM CLASSES OF THE LETTERS IN THIS SENTENCE.

Feb 17 Reading [PS] pgs 44-51

5) _____________Surfaces: Figure out what you get by gluing some surfaces by hand--start with polygons and identify edges. [Link to worksheet: http://www.math.uakron.edu/~sf34/class_home/topo/gluing.pdf .]

Feb 24 Reading [AT] pgs 1-10

6) _____________Euler characteristic, genus: Find the Euler characteristic of a genus-*n* surface with *k* distinct boundary components. Find the Euler characteristic of an *n*-dimensional polytope.

March 2 Homotopy groups. Reading [AT] pgs 21-39

7) _____________Find the fundamental groups of the circle, sphere, torus, Klein bottle, two-holed torus, punctured torus and thrice punctured sphere.

March 9 Reading [AT] pgs 40-55

8) ______________Figure out the fundamental groups for some knot complements: the Knot Groups for the trefoil and the figure eight, and one more.

March 16 Reading [AT] pgs 339-340

9) ______________Higher homotopy: What is the definition of the homotopy group $\pi_2(X, x_0)$. Define $\pi_n(X, x_0)$. Show that $\pi_2(X, x_0)$ is abelian. What is the open problem of homotopy for the spheres?

March 30 Homology groups. Reading [AT] pgs 95-107

10) ______________Find the homology groups of the circle, sphere, torus, Klein bottle and projective plane.

April 6 Reading [AT] pgs 137-143, pgs 5-8

11) ______________Use Simplicial and Cellular complexes, simplicial or cellular homology to calculate homology groups. Find the homology groups of the two-holed torus, punctured torus, thrice punctured sphere and 3-torus.

April 13 Reading [AT] pgs 149-159

12) ______________Calculate (maybe using Mayer-Vietoris) the real homology sequences of: the solid 3d-ball with a toroidal hole (missing a torus-shaped solid), the solid torus with a missing 3-ball, and the solid torus with a toroidal hole.

April 20 Polytopes from topologies. Reading Graph Associahedra pgs 1-8

13) ______________Define graph tubings in terms of topological bases on nodes in a graph. Create 3 examples, on two different graphs, of a graph tubing. List the open sets.

April 27 Spaces of Trees. Reading Tree spaces pgs 9-24

14) ______________Define Billera Holmes Vogt (BHV) space, Balanced Minimal Evolution (BME) polytope

May 4 Spaces of Networks. Reading Network spaces pgs 2-13

15) ______________Define the spaces of networks CSN_n and $\mathfrak{S}$_n