$P^{2} \# P^{2}$


From this picture and those below we can find several fundamental groups.
If just $P^{2}$, we don't have $b$, so $a$ is the only generator and the relation is $a=a^{-1} . \quad\left\langle a \mid a=a^{-1}\right\rangle$
Thus the group is the integers $\bmod 2$, where $1+1=0$.
If $P^{2}-D^{2}$, we have generators $a, b$, and relation $b a=a^{-1}$.

$$
\left\langle a, b \mid b a=a^{-1}\right\rangle
$$

If $P^{2} \# P^{2}$, we have generators $a, b, \mathrm{c}$ and relations $b a=a^{-1}$ and $b c=c^{-1}$.
Exercise: We know that $P^{2} \# P^{2}=K^{2}$ so find an isomorphism from this group

$$
\left\langle a, b, c \mid b a=a^{-1}, b c=c^{-1}\right\rangle
$$ to the one we found in class!


$P^{2}-D^{2}$


