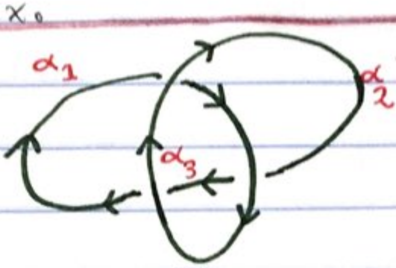
The knot group detects the unknot.

$U=$  $\pi_1(\mathbb{R}^3 - U) = \langle x_1 \mid \phi \rangle \cong \mathbb{Z}$

Example: α_1, α_2, α_3

$K=$ 

α_2 is the entire arc: 

α_3 is very short ←

$$\pi_1(\mathbb{R}^3 - K) = \langle x_1, x_2, x_3 \mid x_2 x_1 x_2^{-1} = x_2,\ x_2 x_2 x_2^{-1} = x_3,\ x_2 x_3 x_2^{-1} = x_1 \rangle$$

$$= \langle x_1, x_2, x_3 \mid x_2 x_1 = x_2^2,\ x_2 = x_3,\ x_2 x_3 = x_1 x_2 \rangle$$

$$\cong \langle x_1, x_2 \mid x_2 x_1 = x_2^2,\ x_2^2 = x_1 x_2 \rangle$$

$$= \langle x_1, x_2 \mid x_1 = x_2,\ x_2 = x_1 \rangle$$

$$= \langle x_1 \mid \phi \rangle$$

$$\cong \mathbb{Z},\quad = \pi_1(\mathbb{R}^3 - U)$$

Theorem: $\pi_1(\mathbb{R}^3 - K) \cong \mathbb{Z}$ iff $K \sim U$.

Here, $\sim$ is knot equivalence, which basically means the same up to movement of the string. The diagrams are the same after applying the following: Reidemeister moves