Knots K

A closed knot is a loop (closed, non-self-intersecting, path) in $\mathbb{R}^3$. It has an orientation!

A knot diagram is a 2D picture of a knot; it is basically the shadow of the knot K projected on a plane; but we draw the intersections (crossings) of the shadow with itself like this:

under

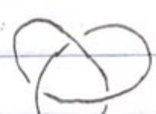
over

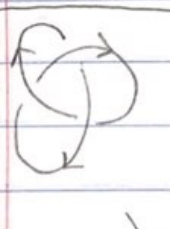
Ex:

$K=$  , the trefoil (left handed) or "overhand" knot

...

Actually: two! ....

$K' =$  (right-handed)

 = 

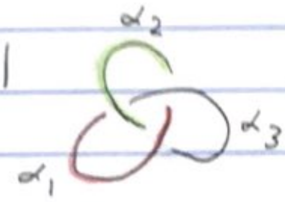
orientation reversed

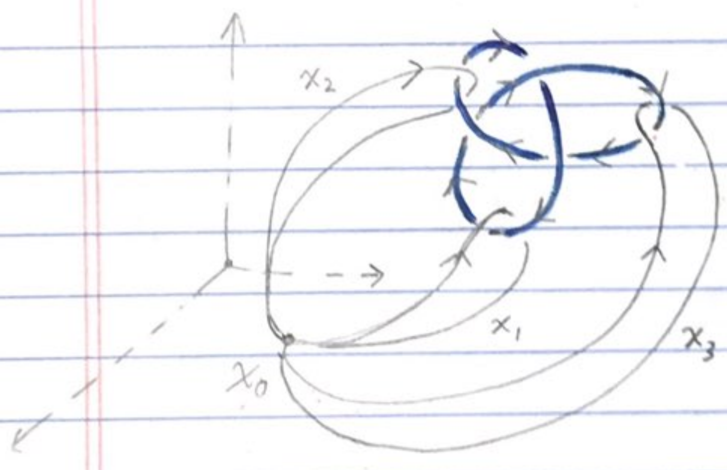
Ex:

the figure eight knot

The knot group of K is the fundamental group of $\mathbb{R}^3 - K$.

$\pi_1(\mathbb{R}^3 - K)$ K = trefoil





For any knot K

Given x_0,

Let generators be $x_1, x_2, x_3, \ldots$

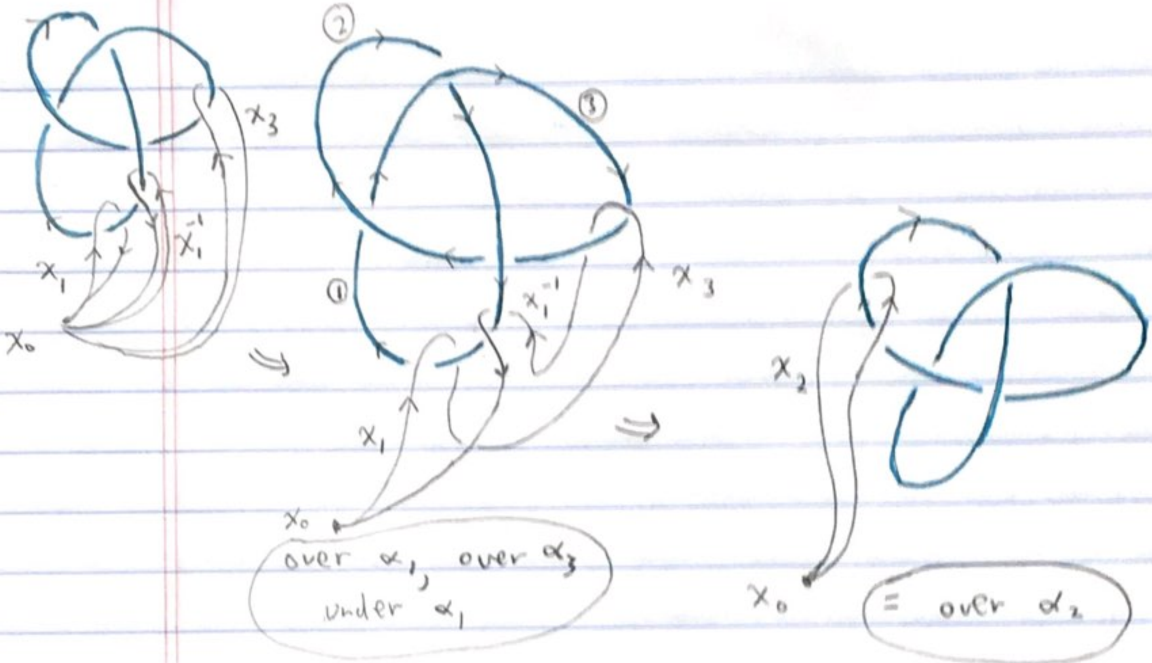
[one x_i for each arc α_i in K.]

where x_i goes <u>over</u> α_i

... then the presentation is...

$\pi_1(\mathbb{R}^3 - K) = \langle x_1, x_2, x_3, \ldots \mid x_1 x_3 x_1^{-1} = x_2 \ \ldots \rangle$

↑ a relation for each crossing!



So for trefoil:

$$\pi_1 = \langle x_1, x_2, x_3 \mid x_1 x_3 x_1^{-1} = x_2,\ x_2 x_1 x_2^{-1} = x_3,\ x_3 x_2 x_3^{-1} = x_1 \rangle$$

at the crossing...
α_1 goes over; α_3 under α_1
to become α_2

$$x_1 = x_2 x_1 x_3^{-1} = x_2^{-1} x_3 x_2 = x_3 x_2 x_3^{-1}$$

$$\Rightarrow \quad x_3 x_2 x_3 = x_2 x_3 x_2$$

$\Rightarrow$

$$\pi_1 \cong \langle a, b \mid bab = aba \rangle$$

[use $\varphi(x_2) = a$, $\varphi(x_3) = b$, $\varphi(x_1) = bab^{-1}$ for the isomorphism φ]