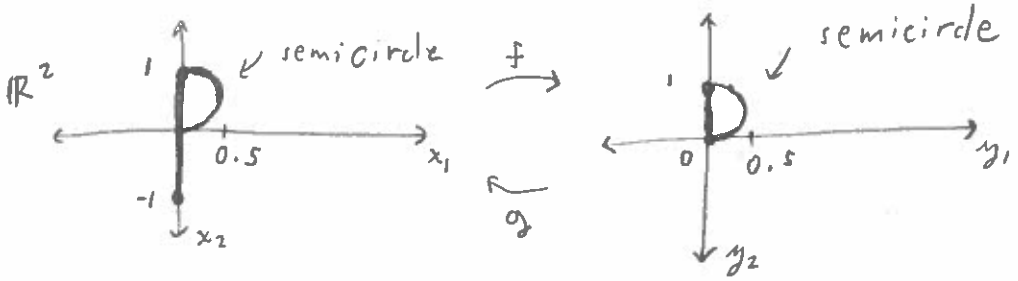
Ex: Show $P \simeq D$, where:



P D

Need $f : P \to D$, $g : D \to P$

$F : P \times I \to P$, $G : D \times I \to D$

$F(\vec{x}, 0) = gf$ $\quad$ $G(\vec{y}, 0) = fg$

$F(\vec{x}, 1) = \mathbb{1}$ $\quad$ $G(\vec{y}, 1) = \mathbb{1}$

Answer $f(\vec{x}) = \begin{cases} \vec{0}, & \text{for } x_1 \leq 0 \text{ ("stem of P")} \\ \vec{x}, & \text{otherwise} \end{cases}$

$g(\vec{y}) = \vec{y}$ (identity, $g = \mathbb{1}$)

$F(\vec{x}, t) = \begin{cases} t\vec{x}, & x_1 \leq 0 \\ \vec{x}, & \text{otherwise} \end{cases}$

$G(\vec{y}, t) = \vec{y}$

Check: $gf : P \to P = f(\vec{x}) = F(\vec{x}, 0)$ ✓ $\left[g(f(\vec{x})) = f(\vec{x}) = \begin{cases} 0\vec{x} \\ \vec{x} \end{cases} \right]$

$fg : D \to D = \mathbb{1} = G(\vec{y}, 0)$ ✓ $[f(g(\vec{y})) = \vec{y}$ since D all above axis$]$

$F(\vec{x}, 1) = \vec{x}$

$G(\vec{y}, 1) = \vec{y}$ $\Big\rangle$ both $= \mathbb{1}$. ✓