Topology project.

Each student has a graph on six nodes, the set of nodes is X = {1,2,3,4,5,6}.

1) Construct 5 distinct bases B1-B5, each with at least 4 sets. Each basis will contain X, and at least 3 more sets. Each set of the basis must be connected in the graph sense: the nodes in that set span a connected subgraph. Each basis should have the property that the edges of the graph, considered as pairs of nodes, are connected in the subspace topology from the basis.

The first two bases B1, B2 should have some sets that overlap: they are not disjoint but neither is contained in the other.

The last three can have nested sets, one contained in the other, but no overlapping otherwise. B3 and B4 should have 6 sets each (including X.) B5 should have 4 sets exactly (X and three more). B5 should have at least one set that is not in either B3 or B4.

Draw your bases as circled subgraphs, and then list them as sets of sets below. For each basis, find the topology by including all unions.

Step 1 is due by March 3.

2) For your bases B3 and B4: Either prove that the topological spaces are homeomorphic or that they are not.

3) The reconnected complement of the graph with respect to a basis element U is found by deleting that set U (and its adjacent edges) and then adding edges to reconnect any two nodes which used to both be attached to that set U with edges. Show that in B1 and B2 there are sets U whose reconnected complement includes edges that are not connected in the original topology. Show that in B5 each set's reconnected complement has edges that are connected in the original topology.

4) For B5, find all the ways of adding 2 more sets while preserving the property just checked in #3. To do this just make sure you follow the rules for B3 and B4—no overlaps unless nested. Note that if you find one way, say B6, and remove any set, there is a unique different set to replace it with. Arrange the answers (via names B5a, B5b, etc.) as corners of a polygon with edges of the polygon showing this remove-replace operation.

5) For any two of your bases, find the fundamental group. Either show that those two bases are homotopy equivalent or that they are not.

Steps 2-5 are due by April 2.

6) For your B1-B5, add an arrow to each edge of the graph, if it can point from inside any basis set to outside that same basis set. There is never an edge pointing both ways, since the edges must be connected as subspaces, so the two nodes will never be inside two separate basis sets.

Check your five results: Do the arrows ever result in a cycle, where following the arrows returns to a previously visited node?________________

What do you call a field of arrows that never makes a circular path, in Calculus 3?____________

Now, in B2, B3, and B5: choose 1 set (not X), find the reconnected complement with the subspace topology, and try to add arrows to the new graph. When is it possible to add arrows to all the edges?

Choose any one of the following 4 questions. The answers are open ended, but 1-2 pages is about right.

7) Application (creative). Let nodes be 6 different cities where an infectious virus has been diagnosed, and the edges routes of direct travel. Let the basis sets be clusters of cities with nearly the same sick population sizes (or larger) connected by travel routes. What are some ways to use this model for helping to monitor and mitigate pandemics? Create a couple of examples. Is the topological model useful or not so much---that is, does it seem to fit or are there common real-world situations where the topological model would fail?

8) Application (creative). Let the arrows (from #6) show the flow of power on the electric power grid. Notice that the basis sets are self-sufficient collections: they don't need any outside power. What are some ways to use this model for monitoring or improving the power grid, and is there any advantage to the extra rules for B3, B4, B5 as opposed to allowing overlaps like in B1, B2? Create a couple of examples. Is the topological model useful or not so much---that is, does it seem to fit or are there common real-world situations where the topological model would fail?

9) Pure (creative). Is there any formula for the number of possible bases on a graph obeying the B3, B4 rules? Conjecture using the numbers of edges, nodes, cycles of your graph. Test the conjecture with 2 new graphs.

10) Pure (creative). Find the homology groups of your graph. Conjecture how the fundamental group for the bases of type B3 and B4 may depend on the graph structure. Test the conjecture with 2 new graphs.

Steps 6, and one of 7-10, are due by May 5.