Sample project, alternate graph, parts 2-5

2/23/20

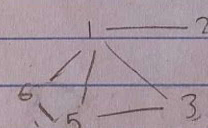
2) Looking at Bases B3, B4, we can define the map:

$f: B3 \to B4$
$\{1\} \to \{4\}$
$\{2\} \to \{3\}$
$\{3\} \to \{2\}$
$\{4\} \to \{1\}$
$\{5\} \to \{6\}$
$\{6\} \to \{5\}$

and with this, all subsequent sets will be mapped. Since we can define this mapping, and it is 1-1, this finite map becomes a bijection, and as such, a homeomorphism. So $B3 \cong B4$.

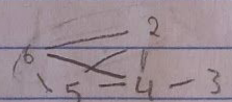
3) B1, consider the set $\mathcal{U} = \{4\} \in B1$.

When we draw the reconnected complement, we have



, which contains some new edges $\{5,6\}$, and $\{3,5\}, \{1,3\}$ but $\{3,5\}$ is disconnected as a subspace of B1.

B2, consider the set $\mathcal{U} = \{1\} \in B2$ when we draw the reconnected complement, we have



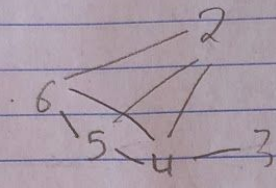
which contains the edge $\{2,6\}$.

But $\{2,6\}$ is disconnected as a subspace of B2.

=>

(3) continued All B5 non-trivial sets, $\{1\}, \{1,5\}, \{1,5,6\}$

Reconnected complement w.r.t. $\{1\}$:

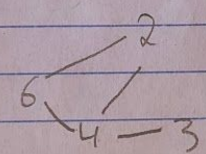


contains new edges:

$\{2,4\}, \{2,5\}, \{2,6\}, \{4,6\}$

$\{2,4\} \cap B5 = \{2,4\}$ connected ✓
$\{2,5\} \cap B5 = \{2,5\}, \{5\}$ connected ✓
$\{2,6\} \cap B5 = \{2,6\}, \{6\}$ connected ✓
$\{4,6\} \cap B5 = \{6\}$ connected ✓

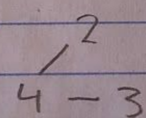
Reconnected complement w.r.t. $\{1,5\}$:



contains new edges:
$\{2,6\}, \{2,4\}, \{4,6\}$

$\{2,4\} \cap B5 = \{2,4\}$ connected ✓
$\{2,6\} \cap B5 = \{2,6\}, \{6\}$ connected ✓
$\{4,6\} \cap B5 = \{4,6\}$ connected ✓

Reconnected complement w.r.t. $\{1,5,6\}$:

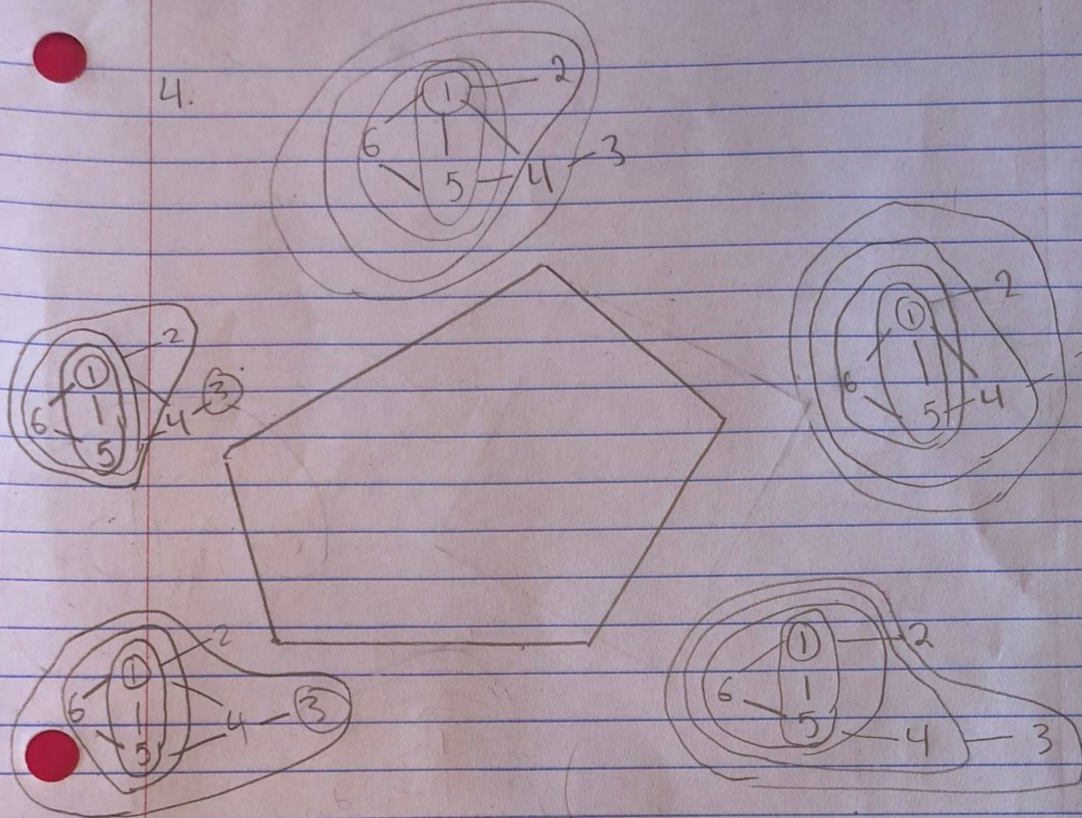


contains new edges:
$\{2,4\}$

$\{2,4\} \cap B5 = \{2,4\}$ connected ✓

So each set's reconnected complement has edges connected as subspaces w.r.t. B5.

4.



5.

For B_3: $\pi_1(X, O_3) = \{e\}$

For B_2: $\pi_1(X, O_2) = \{e\}$