Chp2 Linear combinations

Given several vectors $\vec{x}, \vec{y}, \vec{z}, \vec{w} \ldots$

a linear combination is: choosing scalar multipliers $c_1, c_2, c_3 \ldots$ for each, and then adding them up like this:

$$c_1\vec{x} + c_2\vec{y} + c_3\vec{z} + c_4\vec{w} + \ldots$$

→ We have seen this already: a system of linear equations (with constant term) can be described as a lin. comb. of the coefficient vectors with variable multipliers.

ex)
$$\begin{cases} x_1 + 2x_2 - 3x_3 = 5 \\ 2x_1 \qquad + 4x_3 = 2 \end{cases}$$

equals

$$\left.\underbrace{x_1\begin{pmatrix}1\\2\end{pmatrix} + x_2\begin{pmatrix}2\\0\end{pmatrix} + x_3\begin{pmatrix}-3\\4\end{pmatrix}}_{\text{lin. comb.}} = \begin{pmatrix}5\\2\end{pmatrix}\right\}\text{affine vector equation}$$

→ And we saw it as a way to write the general solution with free variables.

$$\left[\begin{array}{ccc|c}1 & 2 & -3 & 5\\ 2 & 0 & 4 & 2\end{array}\right] \sim \left[\begin{array}{ccc|c}1 & 2 & -3 & 5\\ 0 & -4 & 10 & -8\end{array}\right] \sim \left[\begin{array}{ccc|c}1 & 0 & 2 & 1\\ 0 & 1 & -5/2 & 2\end{array}\right]$$

$$\Rightarrow \left.\begin{array}{l} x_1 + 2x_3 = 1 \\ x_2 - 5/2\, x_3 = 2 \\ x_3 = x_3 \end{array}\right\} \quad \left.\begin{array}{l} x_1 = -2x_3 + 1 \\ x_2 = \frac{5}{2}x_3 + 2 \\ x_3 = x_3 \end{array}\right\} \quad \vec{x} = \underbrace{x_3\begin{pmatrix}-2\\5/2\\1\end{pmatrix} + \begin{pmatrix}1\\2\\0\end{pmatrix}}_{\text{lin. comb.}}$$