4) Matrix transpose: $A_{m\times n} \rightarrow A^t_{n\times m}$

$$(A^t)_{ij} = A_{ji}$$ "rows become columns"

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 0 \end{bmatrix} \rightarrow A^t = \begin{bmatrix} 3 & 4 \\ 2 & 5 \\ 1 & 0 \end{bmatrix}$$

5) Matrix determinant: $A_{n\times n} \rightarrow \det(A) \in \mathbb{R}$

(square A) → scalar,

$$\det A = \sum_{j=1}^{n} (-1)^{j+1} A_{1j} \det(M_{1j})$$

where M_{1j} is the matrix made from A by deleting row 1 and column j.

→ Recursive! We also need: $\det(c) = c$ for any scalar c, which is a 1×1 matrix.

$$\det \begin{bmatrix} 3 & 1 & 2 \\ 4 & 5 & 6 \\ 0 & -3 & -1 \end{bmatrix} \quad \left\{ \begin{array}{l} n=3 \\ k = 1 \text{ to } 3 \end{array} \right.$$

$$= (-1)^2\, 3 \det\begin{bmatrix} 5 & 6 \\ -3 & -1 \end{bmatrix} + (-1)^3\, 1 \det\begin{bmatrix} 4 & 6 \\ 0 & -1 \end{bmatrix} + (-1)^4\, 2 \det\begin{bmatrix} 4 & 5 \\ 0 & -3 \end{bmatrix}$$

$$= 3\left((-1)^2 5 \det(-1) + (-1)^3 6 \det(-3)\right)$$
$$+ -1\left((-1)^2 4 \det(-1) + (-1)^3 6 \det(0)\right)$$
$$+ 2\left((-1)^2 4 \det(-3) + (-1)^3 5 \det(0)\right)$$

$$= 3(5(-1) - 6(-3)) - (4(-1) - 6(0)) + 2(4(-3) - 5(0))$$

$$= 3(13) + 4 + 2(-12)$$

$$= \boxed{19}$$

- Using any row i

$$\det A = \sum_{j=1}^{n} (-1)^{i+j} A_{ij} \det(M_{ij})$$

going along row i

$(-1)^{i+j}$ has checkerboard pattern

$$\begin{bmatrix} + & - & + & - & + & \cdots \\ - & + & - & + & - & \\ + & - & + & - & + & \\ \vdots & & & & & \ddots \end{bmatrix}$$

(odd + odd = even)

- or you can use a column!

- So, if A has a row of zeros, or a column of zeros, then $\det(A) = 0$.

- If A is triangular (either all zeros above or below the main diagonal (upper left to lower right) then $\det(A)$ = multiplying all the main diagonal entries A_{ii}.

$$\det \begin{bmatrix} 3 & 0 & 4 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -4 \end{bmatrix} = -24$$