Matrix operations : useful for short cuts.

1) Matrix times vector $A_{m \times n}$ times $\vec{x} \in \mathbb{R}^n$; $A\vec{x} \in \mathbb{R}^m$

→ mutiply components and sum (dot product)

for each row of A (length n) and all of $\vec{x}$.

ex.

$A_{2\times4}$, $\vec{x} \in \mathbb{R}^4$

$$\underset{A}{\begin{bmatrix} 1 & 3 & 0 & 2 \\ 5 & 4 & -1 & 0 \end{bmatrix}} \underset{\vec{x}}{\begin{pmatrix} 3 \\ 2 \\ 1 \\ -1 \end{pmatrix}} = \begin{pmatrix} 3+6+0-2 \\ 15+8-1+0 \end{pmatrix} = \begin{pmatrix} 7 \\ 22 \end{pmatrix}$$

$A\vec{x} \in \mathbb{R}^2$

2) Matrix times matrix

$A_{m \times n}$ times $B_{n \times q}$ gives AB, $m \times q$.

→ find the entries of AB (in say row i and column j) by multiplying and summing (dot product) row i of A times column j of B.

formula: $(AB)_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$

ex: $\begin{bmatrix} 3 & 0 & 1 & 2 \\ 4 & -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$

$A_{2\times 4}$ ↗ $B_{4\times 3}$ ↗

$$\underset{(2\times 3)}{AB} = \begin{bmatrix} 3+0+1+2 & 0+0-1+0 & 6+0+0-2 \\ 4+0+0+2 & 0-1+0+0 & 8-1+0-1 \end{bmatrix} = \begin{bmatrix} 6 & -1 & 4 \\ 6 & -1 & 6 \end{bmatrix}$$

3) Matrix + Matrix, scalar times matrix

A, B both $m \times n$

$(A+B)_{ij} = A_{ij} + B_{ij}$

$(cA)_{ij} = c(A_{ij})$

$$\begin{bmatrix} 3 & 2 & 0 \\ 4 & 1 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 4 & 3 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 3 \\ 3 & 2 & -2 \end{bmatrix}$$

$$5\begin{bmatrix} 3 & 2 & 0 \\ 4 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 10 & 0 \\ 20 & 5 & -10 \end{bmatrix}$$