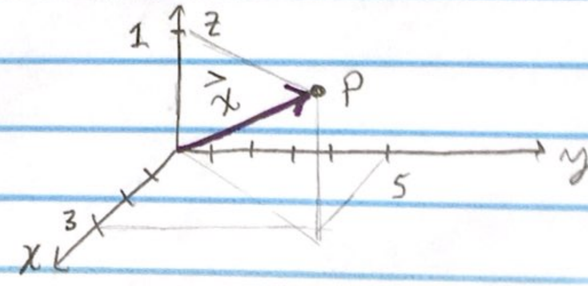
Points & Vectors in $\mathbb{R}^n$

Any point in $\mathbb{R}^n$ can also be written as a (thought of as) a vector in $\mathbb{R}^n$.



$\mathbb{R}^3$ point $P = (3, 5, 1)$

vector $\vec{x} = \langle 3, 5, 1 \rangle = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$

Points are better for describing location, so the numbers are called coordinates.
Vectors also describe location, but can also describe moving in that direction, or a force pulling in that direction, so the numbers are called components.
We can add components to add vectors, and scale vectors by multiplying components.

$$2\begin{pmatrix}3\\5\\1\end{pmatrix} = \boxed{\begin{pmatrix}6\\10\\2\end{pmatrix}}$$

$$\begin{pmatrix}3\\5\\1\end{pmatrix} + \begin{pmatrix}0\\-3\\6\end{pmatrix} = \boxed{\begin{pmatrix}3\\2\\7\end{pmatrix}}$$

Recall dot product $\langle 0,-3,6\rangle\cdot\langle 3,5,1\rangle = 0-15+6=\boxed{-9}$

With variables: $\langle 3,5,1\rangle\cdot\langle x,y,z\rangle = \boxed{3x+5y+1z}$

Rows of coefficients $\begin{bmatrix}3 & 5 & 1\\2 & 0 & -4\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-9\\2\end{pmatrix}$

is a way to write the system

$$\begin{aligned}3x+5y+z&=-9\\2x-4z&=2\end{aligned}$$

so solve $\left[\begin{array}{ccc|c}3 & 5 & 1 & -9\\2 & 0 & -4 & 2\end{array}\right]$ $R_1 \leftarrow R_1 - R_2$ $\left[\begin{array}{ccc|c}1 & 5 & 5 & -11\\2 & 0 & -4 & 2\end{array}\right]$

$R_2 \leftarrow R_2 - 2R_1$ $\left[\begin{array}{ccc|c}1 & 5 & 5 & -11\\0 & -10 & -14 & 24\end{array}\right]$ $\begin{array}{l}R_2 \leftarrow R_2/-10\\R_1 \leftarrow R_1 - 5R_2\end{array}$ $\left[\begin{array}{ccc|c}1 & 0 & -2 & -1\\0 & 1 & 1.4 & -2.4\end{array}\right]$

$$\Rightarrow \quad \begin{aligned} x_1 - 2x_3 &= 1 \\ x_2 + 1.4x_3 &= -2.4 \\ x_3 &= x_3 \quad \text{(free!)} \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} x_1 &= 1 + 2x_3 \\ x_2 &= -2.4 - 1.4x_3 \\ x_3 &= x_3 \end{aligned}$$

OR

$$\boxed{\begin{aligned} x &= 1 + 2z \\ y &= -2.4 - 1.4z \\ z &= z \quad \text{(free!)} \end{aligned}}$$

general

specific

$$\boxed{\begin{aligned} x &= 1 \\ y &= -2.4 \\ z &= 0 \end{aligned}}$$

pick any value

OR

$$\boxed{\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = z\begin{pmatrix} 2 \\ -1.4 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2.4 \\ 0 \end{pmatrix}}$$

this version of the general answer for a system is called a linear combination of constant vectors. with one variable coefficient. In general there is one vector for each free variable plus one constant vector (no variable).