→ Other possibilities :

- only one unique solution : when every column is a pivot column, and any row in B of "0"s ends in an augment of 0 in that row.
- Zero solutions : when there is a row of "0"s in B but the augment column is not 0 in that row.

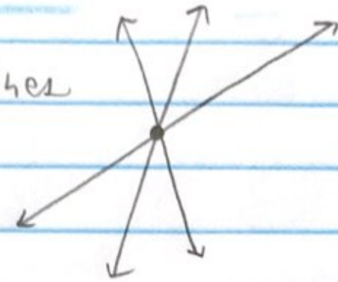
ex:

$$\overbrace{\begin{matrix} 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}}^{B} \left|\begin{matrix} 0 \\ 3 \\ 5 \end{matrix}\right.$$

$0 = 5$ $\boxed{\text{no solution}}$

So now we know some facts to conclude:

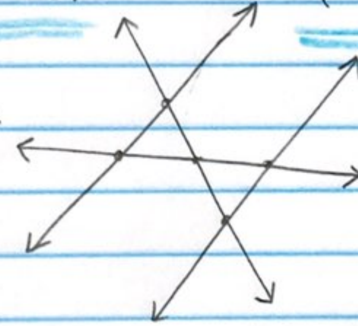
This set of lines in $\mathbb{R}^2$



has only one solution (x, y) So...

...it has a matrix $A_{3\times 2}$, (3 equations, 2 variables)
3 ↑rows 2 ↑columns
(with a extra augmented column)
and together they row reduce to r.r.e.f. B, (with augment),
that has 2 pivots (both columns)
and a row of "0"s (with 0 in augment).

This set of lines in $\mathbb{R}^2$



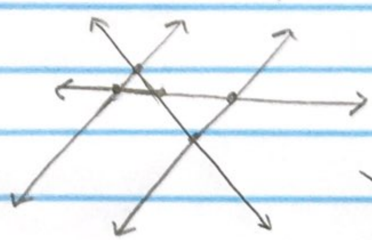
has no solutions!

... So it has a matrix $A_{4\times 2}$ (4 equations, 2 vars)
(with an extra augment column)
which row reduces to r.r.e.f. B,
(with augment)
that has at least one row of "0"s, with a nonzero entry in the augment of that row.

... And, it does have 2 pivots. Why? Just pick two crossing lines to be two rows. One solution!

How to: Reverse engineer, to write a quiz!

Create a system that gives the picture-type:



augmented B: two pivots, random augment

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \\ 0 & 0 & 5 \end{array}\right]$$

Notice: no set of 3 lines here has a solution!

do some random row reduction:

$R1 \leftarrow R1 + 2 \cdot R2$

$R3 \leftarrow R3 - 3 \cdot R2$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 1 & 2 \\ 0 & -3 & -2 \\ 0 & 0 & 5 \end{array}\right]$$

$R2 \leftarrow R2 + R1$

$R4 \leftarrow R4 - R1$

$R3 \leftarrow R3 + R1$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 1 & 3 & 9 \\ 1 & -1 & 5 \\ -1 & -2 & -2 \end{array}\right] \Rightarrow \begin{cases} x + 2y = 7 \\ x + 3y = 9 \\ x - y = 5 \\ -x - 2y = -2 \end{cases}$$

System!

Now, solve that system the usual way, for practice.