→ Geometric meaning and 3 types of solution.

For any number of linear equations,
with any number of variables,
there can only be one of 3 possibilities
→ Zero solutions
→ One solution
→ ∞ solutions

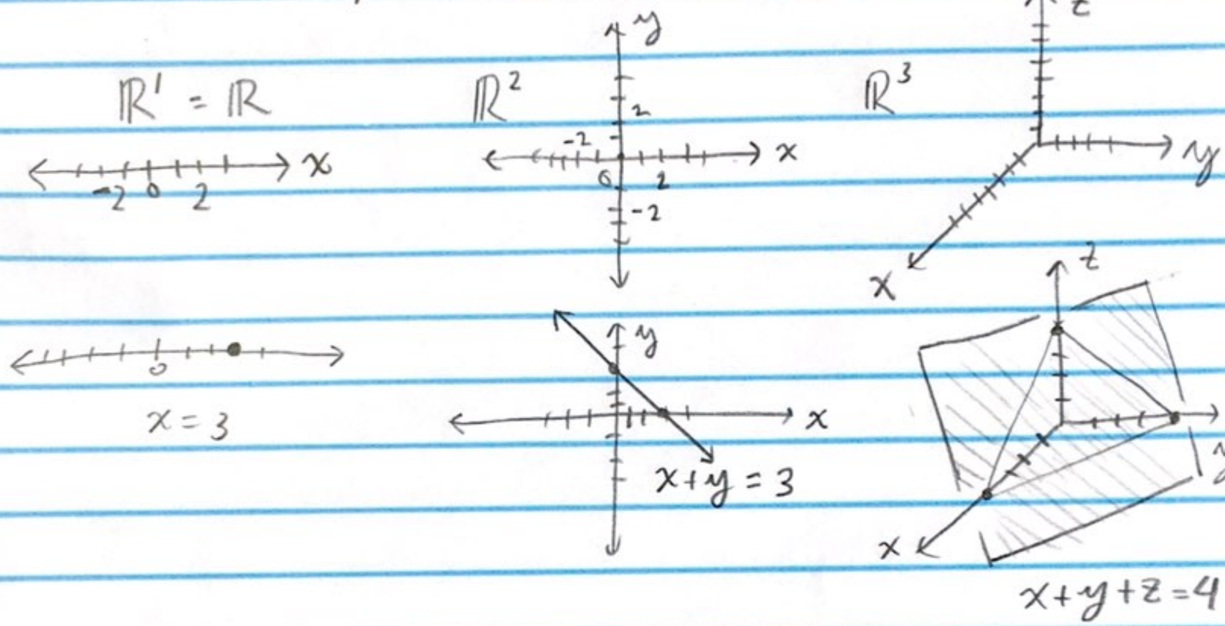
Why?

1) dimension of a space
is a counting number 0, 1, 2, 3, 4, 5, 6, ...
which describes a collection of
points. It tells:
→ the number of independent, free
decisions for perpendicular motions
→ the number of real numbers needed
to describe a single point location
in that space

degrees of freedom

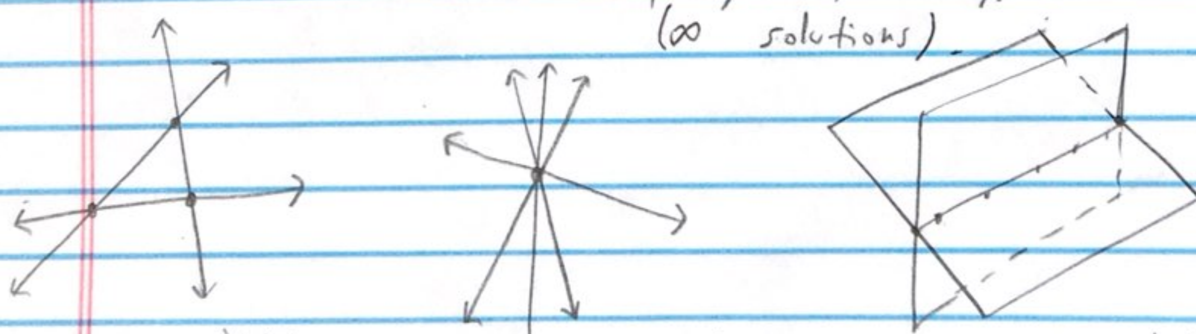
coordinates, # axes R

2) Each single (affine) linear equation describes the points in a <u>hyperplane</u> of the space $\mathbb{R}^d$



3) Several hyperplanes in $\mathbb{R}^d$ can either:

- → not share a common point of intersection (0 solutions)
- → share exactly one common point (needs at least d hyperplanes but no guarantee)
- → OR, all intersect in a lower-dimensional plane, line, or hyperplane (∞ solutions)



no point in common - no solution

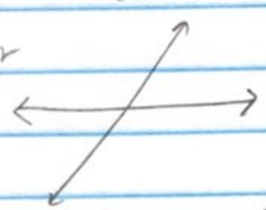
one point solution

line of solution points → ∞ solutions

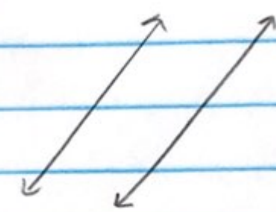
Goal # 1 : be able to look at equations & know what the picture is, and vice versa: look at the picture and know things about the equations.

Two lines in $\mathbb{R}^2$:

either



OR



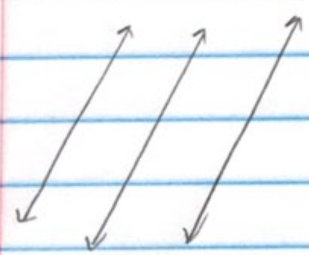
crossing = different slopes | parallel = same slope

ex: $y = \frac{2}{3}x + 1$, $y = \frac{2}{3}x - 5$

$3y - 2x = 1$, $3y - 2x = -5$

slope = coefficients

Three lines in $\mathbb{R}^2$:



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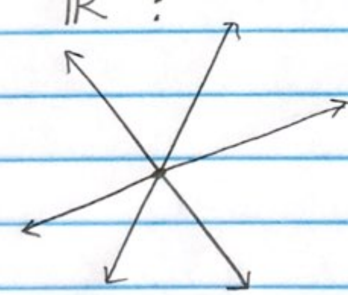

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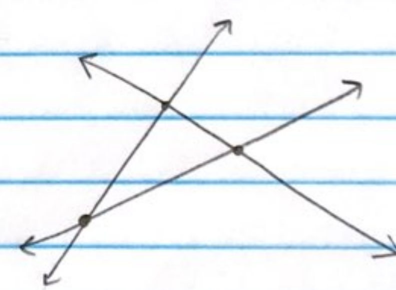

Four lines: 9 different pictures

Five lines: 47

Six lines: 791

Seven lines: 37,830

Eight lines: 4,134,940

Nine lines: Unknown

In general n lines? → open research question.