ex) For any matrix $A_{m\times n}$ the solution to $A\vec{x} = \vec{0}$ is a subspace of $\mathbb{R}^n$.

- the solution contains $\vec{0}$.

We call this solution the Null Space $N(A)$.

ex) Find the null space $N(A)$ for

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

augment

same as solve $A\vec{x} = \vec{0}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \left.\begin{matrix} x_1 = 0 \\ x_2 = x_2 \\ x_3 = 0 \end{matrix}\right\} \quad \vec{x} = x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow N(A) = \boxed{\text{Span}\left\{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}}$$

e) Find the null space $N(B)$ for $B = \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & 3 & 6 & 0 & -3 \end{bmatrix}$

$$\boxed{N(B) = \text{Span}\left\{\begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right\}}$$

solve

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 3 & 6 & 0 & -3 & 0 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 0 & -3 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 & -1 & 0 \end{array}\right]$$

$$\Rightarrow \left.\begin{matrix} x_1 - 3x_3 + 3x_5 = 0 \\ x_2 + 2x_3 - x_5 = 0 \\ x_3 = x_3 \\ x_4 = x_4 \\ x_5 = x_5 \end{matrix}\right\} \left.\begin{matrix} x_1 = 3x_3 - 3x_5 \\ x_2 = -2x_3 + x_5 \\ x_3 = x_3 \\ x_4 = x_4 \\ x_5 = x_5 \end{matrix}\right\} \vec{x} = x_3 \begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$