Spring 2024 Linear Algebra 3450:312

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OFFICE HOURS: MoTuWe 2:45pm-3:45 pm. Lots more by appointment!

Text and Coverage: "Introduction to Linear Algebra with Applications."
DeFranza and Gagliardi. Google play ebook:
https://play.google.com/books/reader?id=LZi3BgAAQBAJ&pg=GBS.PR8&hl=en
or find on Amazon to rent (ISBN 9781478628309).

Website for schedule, homework problems and announcements:
https://sforcey.github.io/sf34/class_home/linear/linears24.htm

Learning Outcomes for 3450:312 Linear Algebra

Students are expected to be able to

- Determine if a set of vectors is linearly independent
- Represent systems of linear equations using matrices and solve such systems
- Find bases and dimensions of vector spaces
- Determine properties of matrix transformations
- Find eigenvalues and eigenvectors for given matrix
- Write elementary proofs

GRADING POLICY:

- The quiz/homework average will be calculated by dropping a total of 15 raw quiz points which means that I'll calculate your percentage by first adding up to 15 points back on to your raw score, limited by the maximum number of hw/quiz points possible. This will have the effect of making a 100% quiz average possible despite some missed homeworks/quizzes.
- There will be 2 in-class closed book tests and the final exam during the semester over the material from lectures, homework and the book. No test may be taken early or late.
- 1000 points possible. For each of these three categories the fraction of points you receive is the same fraction that you earn out of the total possible. If you get 4/5 of the problems correct on test 1, you earn (4/5)*300 = 240 points.

100 pts: Homework, quizzes (10%)
600 pts: 2 Tests at 300 pts each. (60%)
300 pts: Final Exam (30%)

900 pts. guarantees an A
800 pts. guarantees a B
700 pts. guarantees a C
600 pts. guarantees a D
(+,- at my discretion)

Course Outline:

- Jan. 15: No class on MLK day.
- Jan. 17: Day one.
- Chapter 1: Systems of Equations and Matrices
- Chapter 2: Linear Independence
- Jan. 29: Last day to drop.
- Feb. 20: No class, Pres. day.
- TEST 1.
- Chapter 3 : Vector Spaces
- Mar. 3: Last day to w/draw.
- Chapter 4: Linear Transformations
- Chapter 5: Eigenvalues
- TEST 2.
- Mar. 25-31: Spring break.
- Chapter 6: Inner products
- May 3: Last day.
- Final Exam

No notes, formula sheets or books may be used on the any test or the final exam.

Homework may not be copied, but collaboration and research are allowed. All other work is individual. Any incidence of academic dishonesty carries a minimum penalty of a non-removable zero for that work. No active cellular phones, pagers, media players, computers or other electronic communication devices are permitted during the tests.

For information on *"WHAT STUDENTS NEED TO KNOW,"* go to What Students Need To Know : The University of Akron, Ohio (uakron.edu) (see the list of items below).

- The Student Code of Conduct and academic misconduct
- Statement about the ethical use of ChatGPT and other AI tools
- Inclusive Excellence
- Title IX
- Sexual harassment and sexual violence
- Students with disabilities
- Religious accommodations for students
- ZipAssist

Glossary of the main terms: 1-line layman's definitions

-*scalar*: a number. For us, for now, it is a real number c in $\mathbb{R}$ (but complex numbers would also work).
-*linear equation*: summed scalar variables, each multiplied by a scalar coefficient, set equal to a constant scalar.
-*affine* linear equation: when the constant scalar (non-variable portion) is non-zero.
-*system* of linear equations: several linear equations to be solved simultaneously.
-*matrix A*: a rectangular array of scalars, mxn (rows x columns). *Transpose:* A^t means make rows into columns.
-*augmented matrix*: matrix of coefficients with final column the constant scalars (non-variable portions).
-*row reduction moves*: switch two, scale one, or replace a row with a sum of itself and another scaled row.
-*row equivalence*: two matrices are row equivalent $A\sim B$ if related by row reduction moves.
-*pivot*: a 1 in a row of A with all 0's before it in its row and all 0's above and below it in its column.
-*r.r.e.f. of A*: $\sim A$ but each row is either all 0's or has a pivot, and the pivots are as far up and left as possible.
-*pivot column*: a column of A that (eventually) has a pivot in the row reduced echelon form (r.r.e.f.) of A.
-*vector* $\boldsymbol{x}$: anything that can be added to others like it, or multiplied by a scalar (these operations obey rules.)
-*0-vector* $\boldsymbol{0}$: the additive identity vector, which when added to any other vector does not change it.
-*column vector* $\boldsymbol{x}$: nx1 list of scalars, including the usual Calculus 3 vectors in $\mathbb{R}^3$, but also any dimension, $\mathbb{R}^n$.
-*column vector component*: scalar entry in a column vector, also called a coordinate.
-*dimension of a vector* $\boldsymbol{x}$: the number of components in $\boldsymbol{x}$. But the *dimension of a single point* is 0.
-*matrix multiplication*: the entries of AB are summed products (dot products) of the rows of A and columns of B.
-*identity matrix I:* the nxn matrix with 1's on the main diagonal (upper left to lower right) and 0's elsewhere.
-*inverse matrix* A^{-1}*:* when it exists, the nxn matrix that multiplies nxn matrix A on either side to get I.
-*invertible matrix A*: when the inverse of this nxn matrix A exists.
-*singular A*: when the inverse of this nxn A does not exist, non-invertible.
-*minor M of A*: smaller matrix M found by deleting some rows or columns of A. (Sometimes refers to det(M)).
-*determinant of a 1x1 matrix*: det[c] = c, the scalar entry in that matrix.
-*determinant of* nxn *A*: alternating sum using a row of A, each entry times determinant of corresponding minor.
-*linear combination*: sum of vectors, each first multiplied by a scalar.
-*vector space V*: collection of vectors that includes any linear combination of vectors in the collection.
-*dimension of a space V*: number of variable scalar components chosen freely to get a point (vector) of V.
-*subspace S*: vector space that is inside a larger vector space. *Affine subspaces* are shifted by a non-$\boldsymbol{0}$ constant $\boldsymbol{x}$.
-*matrix equation*: matrix times a variable vector, set equal to a constant vector: $A\boldsymbol{x}=\boldsymbol{b}$.
-*vector equation*: linear combination using variable scalars, set equal to a constant vector.
-*homogeneous* vector equation: a linear combination using variable scalars, set equal to the 0-vector.
-*span* of a vector set: the collection of all linear combinations of vectors in that set, always a subspace.
-*linearly independent* vector set: the only linear combination equal to the 0-vector is the one with all scalars = 0.
-*linearly dependent* vector set: linear combinations can equal the 0-vector even with some scalars not 0.
-*linear transformation T*: function from one vector space to another, respects addition and scalar multiplication.
-*domain V of T*: the input vector space.
-*codomain W of T*: the output vector space.
-*range of T*: the actual outputs of T as a subspace of the codomain.
-*kernel of T*: the inputs of T which all output (get sent to) the 0-vector, as a subspace of the domain, *null space.*
-*one-to-one T*: two distinct inputs always get sent to two distinct outputs.
-*onto T*: the range is equal to the entire codomain.
-*rank*: the number of pivot columns in a matrix. Dimension of the range.
-*nullity*: the number of non-pivot columns in a matrix (number of free variables). Dimension of the kernel.
-*basis* $\mathcal{B}$ *of space V*: linearly independent set of n vectors whose span is the space V. Dimension of V is n.
-*column vector* $\boldsymbol{x}_{\mathcal{B}}$ *with respect to an ordered basis* $\mathcal{B}$*:* the scalars used to form $\boldsymbol{x}$ as a linear combination of $\mathcal{B}$.
-*matrix representative A of T*: columns of A are the outputs of T for a basis of V, with respect to a basis of W.
-*similarity:* nxn A is similar to nxn B when there is an invertible nxn P with $A = PBP^{-1}$.
-*eigenvalue* λ*, eigenvectors* $\boldsymbol{x}$*, and eigenspace* E_λ *of A*: scalar λ and space E_λ with $A\boldsymbol{x} = \lambda\boldsymbol{x}$ for vectors $\boldsymbol{x}$ in E_λ.
-*geometric multiplicity of eigenvalue* λ: dimension of the eigenspace E_λ.