Linear. Quiz 6. Name _____

(1) Consider the following linear transformation $T: \mathcal{P}_2 \to \mathcal{P}_3$, given by $T(f) = 4x^2 f'$. That is, take the first derivative and then multiply by $4x^2$.

(a) Find the matrix for T with respect to the standard bases of \mathcal{P}_n : that is, find $[T]_{\mathcal{E}}^{\mathcal{E}}$, where- $\mathcal{E} = \{1, x, x^2, ..., x^n\}$.

(b) Find N(T) and R(T). You can either work with polynomials or with their coordinate vectors with respect to the standard basis. Write the answers as spans of polynomials.

(c) Find the matrix for T with respect to the alternate bases: $[T]^{\mathcal{B}}_{\mathcal{A}}$, where $\mathcal{A} = \{x - 1, x, x^2 + 1\}, \ \mathcal{B} = \{x^3, x, x^2, 1\}.$

(2) Consider the following linear transformation $T: \mathcal{P}_2 \to \mathcal{P}_3$, given by $T(f) = \frac{d}{dx}(4x^2f)$. That is, multiply by $4x^2$ and then take the first derivative.

(a) Find the matrix for T with respect to the standard bases of \mathcal{P}_n : that is, find $[T]_{\mathcal{E}}^{\mathcal{E}}$, where- $\mathcal{E} = \{1, x, x^2, ..., x^n\}$.

(b) Find the matrix for S(f) = 8xf, again with the standard bases. Check that your answer to 2a equals the matrix from 1a plus the matrix for S. Why should that be?