Stating the problem: I, II, III, VI really the same problem.	Example: $m=2$, $n=3$	Reduce	Example reduction	Example Answer Statement
I. Solve a system of m linear equations in n variables.	$x_1 + 3x_3 = 4$ $2x_1 + x_2 = 1$ $\longrightarrow$	augmented matrix of coefficients		$x_1 = 4 - 3x_3$ $x_2 = -7 + 6x_3$ x_3 free
II. Solve the corresponding vector equation. ↳ Find $\vec{b}$ as a linear combination of $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$. ↳ Is $\vec{b}$ in Span$\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$?	$x_1\begin{bmatrix}1\\2\end{bmatrix} + x_2\begin{bmatrix}0\\1\end{bmatrix} + x_3\begin{bmatrix}3\\0\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$ $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{b}$	augmented matrix $[\vec{a}_1\ \vec{a}_2\ \vec{a}_3\ \vec{b}]$	$\begin{bmatrix}1&0&3&4\\2&1&0&1\end{bmatrix}$ $R2 \leftarrow R2 - 2R1$ $\begin{bmatrix}1&0&3&4\\0&1&-6&-7\end{bmatrix}$	choose $x_3 = 0$ $x_2 = -7$ $x_1 = 4$; Yes $\vec{b} \in$ Span$\{a_1, a_2, a_3\}$
III. Solve the corresponding matrix equation.	$\underbrace{\begin{bmatrix}1&0&3\\2&1&0\end{bmatrix}}_{A}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$ $A \cdot \vec{x} = \vec{b}$	augmented matrix $[A, \vec{b}]$	$x_1 + 3x_3 = 4$ $x_2 - 6x_3 = -7$ $x_3 = x_3$	$\vec{x} = \begin{bmatrix}4\\-7\\0\end{bmatrix} + x_3\begin{bmatrix}-3\\6\\1\end{bmatrix}$
IV. Solve the corresponding homogeneous vector equation. ↳ Is there a nontrivial solution, which would mean that $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ is linearly dependent?	$x_1\begin{bmatrix}1\\2\end{bmatrix} + x_2\begin{bmatrix}0\\1\end{bmatrix} + x_3\begin{bmatrix}3\\0\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{0}$	augmented matrix $[\vec{a}_1\ \vec{a}_2\ \vec{a}_3\ \vec{0}]$	$\begin{bmatrix}1&0&3&0\\2&1&0&0\end{bmatrix}$ $R2 \leftarrow R2 - 2R1$ $\begin{bmatrix}1&0&3&0\\0&1&-6&0\end{bmatrix}$	$x_1 = -3x_3$ $x_2 = 6x_3$ $x_3 = x_3$ $\vec{x} = x_3\begin{bmatrix}-3\\6\\1\end{bmatrix}$
V. Solve the homogeneous matrix equation. ↳ Are the columns of A lin. dep.?	$A\vec{x} = \vec{0}$ $\longrightarrow$	augmented matrix $[A, \vec{0}]$		Yes, $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ is lin. dep.
VI. Is $\vec{b}$ in the range of $T: \mathbb{R}^n \to \mathbb{R}^m$ given by $\vec{x} \longmapsto A\vec{x}$? ↳ Find $\vec{x}$ whose image under T is $\vec{b}$.	$\overbrace{\begin{bmatrix}1&0&3\\2&1&0\end{bmatrix}}^{2\times 3}\overbrace{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}}^{\mathbb{R}^3} = \overbrace{\begin{bmatrix}4\\1\end{bmatrix}}^{\mathbb{R}^2}$ $A\vec{x} = \vec{b}$	augmented matrix $[A, \vec{b}]$	$\begin{bmatrix}1&0&3&4\\2&1&0&1\end{bmatrix}$ (same as III.)	$\vec{x} = \begin{bmatrix}4\\-7\\0\end{bmatrix} + x_3\begin{bmatrix}-3\\6\\1\end{bmatrix}$