

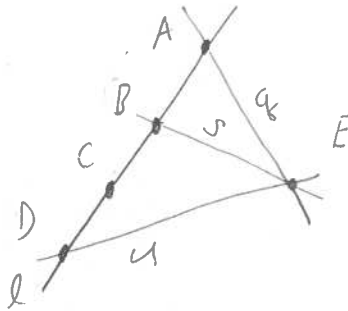
Geometry Test 2 Review: first study quizzes!

Formulas $d_S(A, B) = R \cos^{-1} \left(\frac{A \cdot B}{R^2} \right)$. $d_H(A, B) = \ln \left(\frac{1 - A \cdot B + d_E(A, B)}{1 - A \cdot B - d_E(A, B)} \right)$

Given point set $\mathcal{P} = \{A, B, C, D, E\}$;

- (1) For \mathcal{P} with lines $\mathcal{L} = \{l, q, s, u\}$,
let $\mathcal{I} = \{(A, l), (B, l), (C, l), (D, l), (A, q), (B, s), (D, u), (E, q), (E, s), (E, u)\}$.

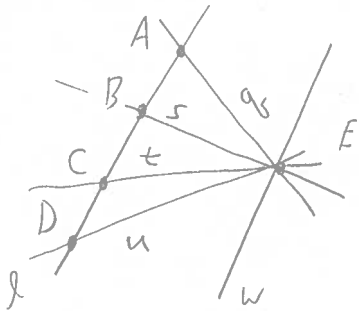
Is this an abstract incidence geometry or not? Draw a diagram and explain.



No,
missing line through
C, E.

- (2) For \mathcal{P} with lines $\mathcal{L} = \{l, q, s, u, t, w\}$,
let $\mathcal{I} = \{(A, l), (B, l), (C, l), (D, l), (A, q), (B, s), (C, t), (D, u), (E, q), (E, s), (E, t), (E, u), (E, w)\}$.

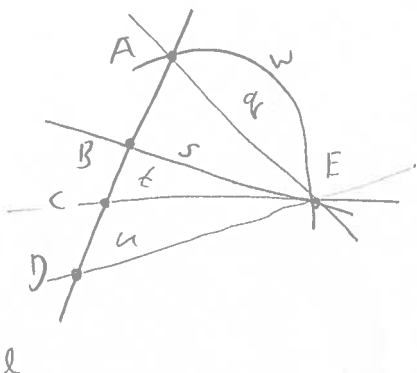
Is this an abstract incidence geometry or not? Draw a diagram and explain.



No,
line w has only
one point.

- (3) For \mathcal{P} with lines $\mathcal{L} = \{l, q, s, u, t, w\}$,
let $\mathcal{I} = \{(A, l), (B, l), (C, l), (D, l), (A, q), (A, w), (B, s), (C, t), (D, u), (E, q), (E, s), (E, t), (E, u), (E, w)\}$.

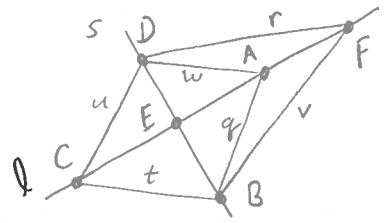
Is this an abstract incidence geometry or not? Draw a diagram and explain.



No,
2 lines w, q through
A, E.

- (4) For $\mathcal{P} = \{A, B, C, D, E, F\}$ with lines $\mathcal{L} = \{l, q, s, u, t, w, r, v\}$,
 let $\mathcal{I} = \{(A, l), (A, q), (A, w), (B, q), (B, s), (B, t), (C, t), (C, l), (C, u), (D, u), (D, s), (D, w), (E, l), (E, s), (F, l), (F, r), (F, v), (D, r), (B, v)\}$.

Is this an abstract incidence geometry or not? Draw a diagram and explain.



Yes, I_1, I_2, I_3
all obeyed.

- (5) For number (4) above, find the line cardinality vector LCV. If another incidence geometry has a different LCV, can you find an isomorphism between them? No

$$\langle 4, 3, 2, 2, 2, 2, 2 \rangle = \langle 4, 3, 6, 2 \rangle$$

- (6) For number (4) above, find the automorphism f such that $f(A) = C, f(B) = B,$ and $f(C) = A$.

x	A	B	C	D	E	F
$f(x)$	C	B	A	D	E	F

- (7) For number (4) above, find the automorphism f such that $f(A) = A, f(B) = D,$ and $f(C) = F$.

x	A	B	C	D	E	F
$f(x)$	A	D	F	B	E	C

- (8) Consider the three points given: $A = (1/2, 0), B = (1/4, 1/4),$ and $C = (1/2, 1/2),$
 Find the 12 distances: Euclidean, Taxicab, Max, Bus, Post-Office, and Hyperbolic between the two points.

$$d_E(A, B) = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \sqrt{\frac{1}{8}} \quad d_E(B, C) = \sqrt{\frac{1}{8}}$$

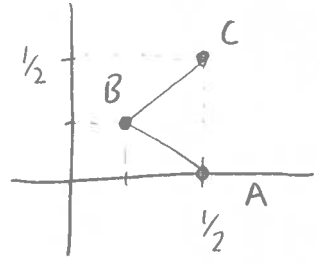
$$d_T(A, B) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \quad d_T(B, C) = \frac{1}{2}$$

$$d_M(A, B) = \frac{1}{4} \quad d_M(B, C) = \frac{1}{4}$$

$$d_B(A, B) = \frac{1}{2} + \sqrt{\frac{1}{8}} \quad d_B(B, C) = \sqrt{\frac{1}{8}}$$

$$d_P(A, B) = \frac{1}{2} + \sqrt{\frac{1}{8}} \quad d_P(B, C) = \sqrt{\frac{1}{8}} + \sqrt{\frac{1}{8}} + \sqrt{\frac{1}{8}} = 3\sqrt{\frac{1}{8}}$$

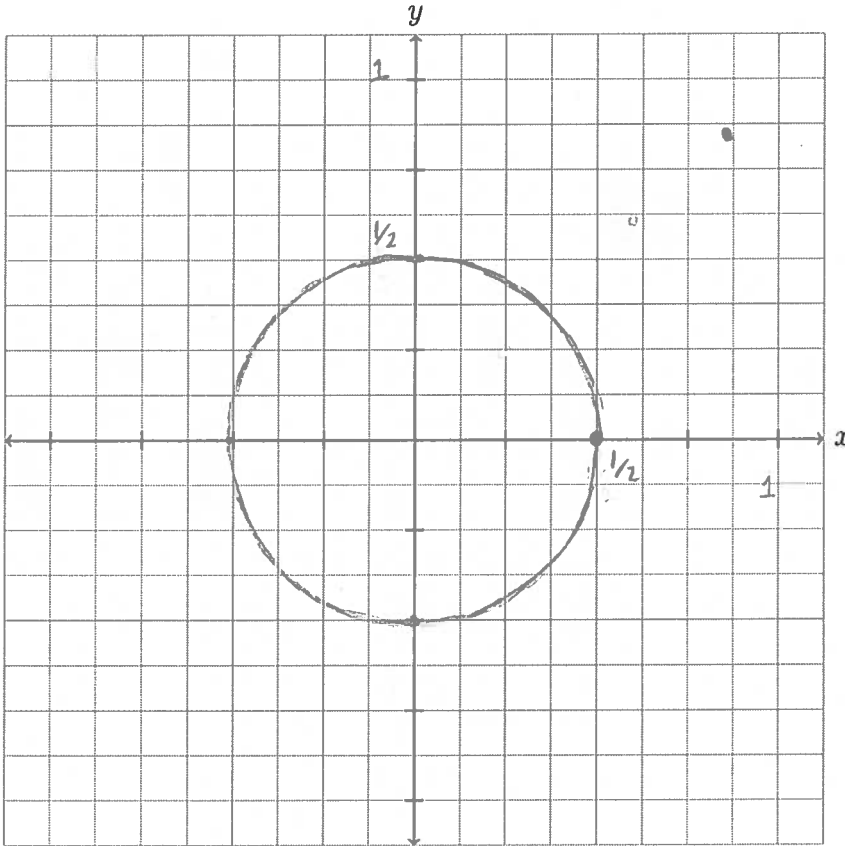
$$d_H(A, B) = \ln\left(\frac{7/8 + \sqrt{1/8}}{7/8 - \sqrt{1/8}}\right) = 0.85 \quad d_H(B, C) = \ln\left(\frac{6/8 + \sqrt{1/8}}{6/8 - \sqrt{1/8}}\right) = 1.02$$



For Euclidean, Taxicab, Max, and Hyperbolic,
what are the equivalence classes of the two segments \overline{AB} and \overline{BC} ?

E	$\{ \{ \overline{AB}, \overline{BC} \} \}$
T	$\{ \{ \overline{AB}, \overline{BC} \} \}$
M	$\{ \{ \overline{AB}, \overline{BC} \} \}$
H	$\{ \{ \overline{AB} \}, \{ \overline{BC} \} \}$

(9) Draw the circle for each metric centered at B through the point A . Use compass and straightedge.



Bus Metric
 d_B

(10) Find the three distances between points $A = (2, 10, 25)$, $B = (2, 14, 23)$, and $C = (7, 14, 22)$ on the sphere with radius = 27.

$$d_S(A, B) = \underline{4.477}, \quad d_S(B, C) = \underline{5.10}, \quad d_S(A, C) = \underline{7.09}.$$

$$27 \cos^{-1} \left(\frac{2 \cdot 2 + 10 \cdot 14 + 25 \cdot 23}{27^2} \right)$$