Geometry Test 1 Review: first study quizzes!

(1) Given universe $\mathcal{U} = \{1, 2, 3, 4, 5, 7, 9, 10, 21, 25\}$; $A = \{7, 9, 10, 21, 25\}$; and $B = \{5, 4, 7, 10, 21\}$. Find the following:

- $\overline{A \cup \overline{B}}$ $= \bar{A} \cap B = B - A = \boxed{\{5, 4\}}$
- $(A - B) \cup (B - A)$ $= \{9, 25\} \cup \{5, 4\} = \boxed{\{9, 25, 5, 4\}}$
- $\overline{(B - A) \cap A}$ $= \overline{(B - A)} \cup \bar{A} = \overline{(B \cap \bar{A})} \cup \bar{A} = \bar{A} = \boxed{\{1, 2, 3, 4, 5\}}$
- $|\mathcal{P}(A)|$ $= 2^{|A|} = 2^5 = \boxed{32}$
- $|\mathcal{P}(A \times B) \times A|$ $= |\mathcal{P}(A \times B)| \cdot |A| = 2^{|A \times B|} \cdot |A| = 2^{|A| \cdot |B|} \cdot |A| = \boxed{2^{25}(5)}$
- $A \cap \overline{A}$ $= \boxed{\phi} = \boxed{\{\}}$
- $\mathcal{U} - \overline{B}$ $= B = \boxed{\{5, 4, 7, 10, 21\}}$

(2) Given $A = \{4, \{5, 7\}, 7, \{7\}, \{\{5\}, 7\}\}$.

- Find $|A|$ $= \boxed{5}$

True or False?

- $\{\{5\}\} \in A$. F
- $\{5\} \in A$. F
- $5 \subseteq A$. F
- $5 \in A$. F
- $7 \in A$. T
- $\{5, 7\} \in A$. T
- $\{7\} \in A$. T
- $\{7\} \subseteq A$. T
- $\{\{7\}, 7\} \subseteq A$. T
- $\{\} \in A$. F
- $\{\} \subseteq A$. T

(3) Given the set $A = \{2, 5, 9\}$ here are some relations on A.

- Consider the relation $R = \{(2,5), (5,2), (5,5), (9,9)\}$

Is R transitive? No reflexive? No symmetric? Yes equivalence? No

- Consider the relation $R = \{(2,5), (2,2), (5,5), (9,9), (2,9)\}$

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- Consider the relation $R = \{(2,2), (5,2), (5,5), (9,9), (2,5)\}$

Is R transitive? Yes reflexive? Yes symmetric? Yes equivalence? Yes

- Consider the relation $R = \{(2,5), (5,2), (5,5), (9,2), (2,9)\}$

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- Consider the relation $R = \{(2,5), (2,2), (5,5), (9,9), (5,9)\}$

Is R transitive? No reflexive? Yes symmetric? No equivalence? No

(4) For the set $S = \{6, 8, 9, 11, 15, 17\}$ Consider the relation
$R = \{(6,6), (8,8), (9,9), (11,11), (15,15), (17,17), (8,9), (9,8), (8,11), (11,8), (9,11), (11,9), (6,15), (15,6)\}$

Is R transitive? Yes reflexive? Yes symmetric? Yes
Since R is an equivalence relation, write R as a partition of S into equivalence classes.

$\{\{6, 15\}, \{8, 9, 11\}, \{17\}\}$

(5) With compass and straightedge:

Construct a copied angle, a perpendicular through a point on and off the line, and a circle through three points.

(in class)

Selected axioms and lemmas for 2D abstract geometry $\mathcal{P}, \mathcal{L}, \mathcal{I}$ with $\mathcal{B}$; choices for proof steps.

1a: $\forall \angle BAC$, and $\overrightarrow{A'B'}$, $\exists!$ C' and C'' on opposite sides of $\overleftrightarrow{A'B'}$ s.t. $\angle B'A'C' \cong \angle BAC \cong \angle B'A'C''$.

1b: If $\angle ABC \cong \angle DEF$ and $\angle DEF \cong \angle GHK$ then $\angle ABC \cong \angle GHK$.

1c: If $\overline{AB} \cong \overline{A'B'}$, $\angle BAC \cong \angle B'A'C'$, and $\overline{AC} \cong \overline{A'C'}$, then $\triangle ABC \cong \triangle A'B'C'$.

2a: $\exists\, l \neq m \in \mathcal{L}$.

2b: $\forall\, l \in \mathcal{L}, \exists A \neq B \in \mathcal{P}$ s.t. both $(A, l) \in \mathcal{I}$ and $(B, l) \in \mathcal{I}$.

2c: $\forall A \neq B \in \mathcal{P}, \exists!\, l \in \mathcal{L}$ s.t. both $(A, l) \in \mathcal{I}$ and $(B, l) \in \mathcal{I}$.

3a: If $l \neq m \in \mathcal{L}$ are not parallel, then $\exists!$ $A \in \mathcal{P}$ s.t. both $(A, l) \in \mathcal{I}$ and $(A, m) \in \mathcal{I}$.

3b: $\forall A \neq C \in \mathcal{P}, \exists B \in \mathcal{P}$ s.t. B is on line $\overleftrightarrow{AC}$ and $(A, B, C) \in \mathcal{B}$.

3c: If $\triangle ABC \cong \triangle A'B'C'$ then all three respective pairs of segments and angles are congruent.

4a: If $\angle BAC = \angle BAD$ then $\overrightarrow{AC} = \overrightarrow{AD}$.

4b: $\forall A \neq B \in \mathcal{P}$, and $(A', l) \in \mathcal{I}$, $\exists!$ B' and B'' with $(B', A', B'') \in \mathcal{B}$ and $\overline{AB} \cong \overline{A'B'} \cong \overline{A'B''}$.

4c: If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$ then $\overline{AB} \cong \overline{EF}$.

4d: If $\overline{AB} \cong \overline{A'B'}$, $\overline{BC} \cong \overline{B'C'}$, and $\overline{AC} \cong \overline{A'C'}$, then $\triangle ABC \cong \triangle A'B'C'$.

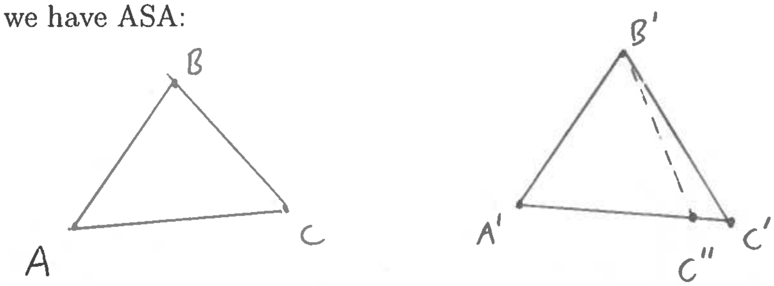
4f: If two circles contain points (on the circles) which are inside the other circle (within the radius), and points (on the circles) that are outside the other, then they intersect in two points.

(6) From the list of axioms and lemmas, select which one justifies each step of the following proof.

Theorem: Given $\mathcal{P}, \mathcal{L}, \mathcal{I}$ a 2D abstract geometry, if A, B, C are three points not all in a line, and A', B', C' are three points not all in a line, and we have ASA:

$\angle BAC \cong \angle B'A'C'$,
$\overline{AB} \cong \overline{A'B'}$,
and $\angle ABC \cong \angle A'B'C'$,

then $\triangle ABC \cong \triangle A'B'C'$.



Proof: The givens imply:

1) $\exists$ unique point C'' on ray $\overrightarrow{A'C'}$ with $\overline{AC} \cong \overline{A'C''}$ — 4b

2) $\Longrightarrow \triangle ABC \cong \triangle A'B'C''$ — 1c

3) $\Longrightarrow \angle ABC \cong \angle A'B'C''$ — 3c

4) $\Longrightarrow \angle A'B'C' \cong \angle A'B'C''$ — 1b

4.5) $\Longrightarrow \angle A'B'C' = \angle A'B'C''$,
since there is only one unique angle on a given side of ray $\overrightarrow{B'A'}$ congruent to $\angle A'B'C'$. — 1a

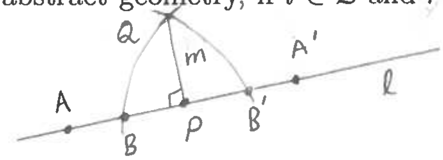
5) $\Longrightarrow \overrightarrow{B'C'} = \overrightarrow{B'C''}$ — 4a

6) $\Longrightarrow \overleftrightarrow{B'C'} \cap \overleftrightarrow{A'C'} = C'$ and
$\overleftrightarrow{B'C'} \cap \overleftrightarrow{A'C'} = C''$, since C'' is on both lines by 1) and 5).
So $C' = C''$ — 3a

7) $\Longrightarrow \triangle ABC \cong \triangle A'B'C'$. — by step 2) with C' replacing C''

(7) From the list of axioms and lemmas, select which one justifies each step of the following proof.

Theorem: Given $\mathcal{P}, \mathcal{L}, \mathcal{I}$ a 2D abstract geometry, if $l \in \mathcal{L}$ and $P \in \mathcal{P}$ with $(P, l) \in \mathcal{I}$, then $\exists m \in \mathcal{L}$ with $(P, m) \in \mathcal{I}$ and $m \perp l$.



Proof: The givens imply:

1) $\exists$ point A on l, with $A \neq P$. 2b

2) $\implies \exists$ point A' on l, with $\overline{AP} \cong \overline{A'P}$, with P between A and A'. 4b

3) $\implies \exists$ point B on l, with B between A and P. 3b

4) $\implies \exists$ point B' on l, with $\overline{AB'} \cong \overline{A'B}$, 4b

5) $\implies$ The circle centered at A through B' will meet the circle centered at A' through B in two points Q, Q'. (Since B, B' are each inside the other circle, and each circle also contains points outside the other.) 4f

5.5) $\implies \overline{AQ} \cong \overline{AB'}$ and $\overline{A'Q} \cong \overline{A'B}$ definition of circle

6) $\implies \overline{AQ} \cong \overline{A'Q}$, by 4) 4c

7) $\implies \exists$ line m through Q and P. 2c

8) $\implies \triangle APQ \cong \triangle A'PQ$. 4d

9) $\implies \angle APQ \cong \angle A'PQ$. 3c

10) $\implies m \perp l$. definition of supplemental and right angles