(1) Prove $\forall a, b \in \mathbb{Z}$, if ($a \mod 6 = 5$ and $b \mod 4 = 3$) then $4a + 6b \mod 8 = 6$.
Use a Direct proof.

a) Write the assumption, translated to algebraic equations.

b) Write what we want to show, translated to algebraic equations.

c) Write the proof steps.

(2) Suppose we were to prove the statement "$\forall y \in \mathbb{Z}$, y is even $\Rightarrow (y^3 - 1)$ is odd." (Answer using alegebraic equations, without using the word "not" or the symbol "$\sim$.")

a)For a direct proof we assume ____________________ and show ____________________.

b) For proof using the contrapositive we assume __________________ and show ____________________.

c) For proof by contradiction we assume ____________________________________ and show that we reach a false conclusion.

(3) Use contradiction to prove: $\forall a, b \in \mathbb{Z}$, if ($a$ is even and b is odd) then 4 does not divide $(a^2 + 2b^2)$.
a) Negate the statement.

b) What do we assume? Translate to algebraic equations.

c) Use the assumptions to prove that $4|2$, as an algebraic equation.

(4) Prove by induction that: $\forall n \in \mathbb{N}$, if $n \geq 2$ then $3|(2^{(4n-4)} + 2^{(2n-3)})$.
a) Show the base case.

b) State the induction assumption, translate to algebraic equations.

c) State what we need to show, translate to algebraic equations.

d) Do the proof steps.

(5) Use a Direct proof to prove: $\forall z \in \mathbb{Z}, 3|(z+1) \Rightarrow z^2 \mod 3 = 1$.

a) Write the assumption, translated to algebraic equations.

b) Write what to show, translated to algebraic equations.

c) Do the proof steps.