Key

Foam Test 1 Review: first study quizzes!

(1) Finish the following truth table. Is the last expression a tautology, contradiction or neither?

P	Q	$\sim Q$	$P \Rightarrow Q$	$P \vee \sim Q$	$(P \Rightarrow Q) \wedge (P \vee \sim Q)$
T	T	F	T	T	T
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

(2) Suppose that P is false and Q is true. Find whether each of these statements is true (T) or false (F).

- $(P \Rightarrow \underbrace{\sim Q}_{F}) \Rightarrow Q$ $= \boxed{T}$

  F T

- $(P \wedge (Q \Longleftrightarrow (\sim P))) \vee Q$ $= \boxed{T}$

  F T

- Repeat the above problems with the alternate given information that P is false and Q is false.

- $\underbrace{(P \Rightarrow \sim Q)}_{T} \Rightarrow Q$ $= \boxed{F}$

  F F

- $(P \wedge (Q \Longleftrightarrow (\sim P))) \vee Q$ $= \boxed{F}$

  F F

(3) Given the statement of implication "($x \in S$ and $x \leq 5$) implies that ($x > 2$ or $x = -10$.)"

- Find its converse; write it without the word "not" and without the symbol "$\sim$."

  $((x > 2) \vee (x = -10)) \Rightarrow ((x \in S) \wedge (x \leq 5))$

- Find its negation; write it without the word "not" and without the symbol "$\sim$."

  $((x \in S) \wedge (x \leq 5)) \wedge (x \leq 2) \wedge (x \neq -10)$

- Find its contrapositive; write it without the word "not" and without the symbol "$\sim$."

  $((x \leq 2) \wedge (x \neq -10)) \Rightarrow ((x \notin S) \vee (x > 5))$

- Find its inverse; write it without the word "not" and without the symbol "$\sim$."

  $((x \notin S) \vee (x > 5)) \Rightarrow ((x \leq 2) \wedge (x \neq -10))$

- If $S = \{3, 4, 7, 11\}$, is the statement true or false for all $x \in S$?

  $\boxed{\text{True}}$

(4) Given the statement: $\forall x \in \mathbb{Z}, (x \text{ even or } x|18) \Rightarrow ((x+1) \text{ is odd and } x^2 > 3)$.

- Find its negation; write it without the symbol "$\sim$."

$\exists x \in \mathbb{Z}$ s.t. $(x \text{ even } \vee x|18) \wedge ((x+1 \text{ even}) \vee (x^2 \leq 3))$.

- Find a counterexample which proves the original statement is false.

$\boxed{x = 1}$

(5) Given the statement: $\forall x \in \mathbb{R}, \exists y \in \mathbb{Z}$ s.t. $yx \leq (yx + x)$.

- Find its negation; write it without the symbol "$\sim$."

$\exists x \in \mathbb{R}$ s.t $\forall y \in \mathbb{Z}$, $yx > (yx + x)$.

(6) Given the statement: If you have a french-apple pie then you have raisins, cherries and a glazed crust.

- Find its contrapositive; write it without the symbol "$\sim$."

If you dont have (raisins, cherries and glazed crust), then you don't have a french apple pie.

- Find its converse; write it without the symbol "$\sim$."

If you have raisins cherries and a glazed crust then you have french apple pie.

- Rewrite the statement using the words "only if."

You have a french apple pie only if you have raisins, cherries and a glazed crust.

- Rewrite the statement using the word "necessary."

Having Raisins, cherries and a glazed crust are necessary for having a french apple pie.

- Rewrite the statement using the word "sufficient."

Having a french apple pie is sufficient for having raisins, cherries and a glazed crust.

(7) Given universe $\mathcal{U} = \{1, 2, 3, 4, 5, 7, 9, 10, 21, 25\}$; $A = \{7, 9, 10, 21, 25\}$; and $B = \{5, 4, 7, 10, 21\}$. Find the following:

- $\overline{A \cup \overline{B}}$ $= \bar{A} \cap B = B - A = \boxed{\{5,4\}}$
- $(A - B) \cup (B - A)$ $= \boxed{\{9, 25, 5, 4\}}$
- $\overline{(B - A) \cap A}$ $= (B-A) \cup \bar{A} = (B \cap \bar{A}) \cup \bar{A} = \bar{A} = \boxed{\{1,2,3,4,5\}}$
- $|\mathcal{P}(A)|$ $= \boxed{2^5}$
- $|\mathcal{P}(A \times B) \times A|$ $= 2^{|A \times B|} \cdot |A| = 2^{5 \cdot 5} \cdot 5 = \boxed{5(2^{25})}$
- $A \cap \overline{A}$ $= \boxed{\emptyset}$
- $\mathcal{U} - \overline{B}$ $= \overline{\overline{B}} = B = \boxed{\{5, 4, 7, 10, 21\}}$

(8) Given $A = \{4, \{5, 7\}, 7, \{7\}, \{\{5\}, 7\}\}$.

- Find $|A|$ $= \boxed{5}$

True or False?

- $\{\{5\}\} \in A$. F
- $\{5\} \in A$. F
- $5 \subseteq A$. F
- $5 \in A$. F
- $7 \in A$. T
- $\{5, 7\} \in A$. T
- $\{7\} \in A$. T
- $\{7\} \subseteq A$. T
- $\{\{7\}, 7\} \subseteq A$. T
- $\{\} \in A$. F
- $\{\} \subseteq A$. T