Foam Test 1 Review: first study quizzes!

(1) Finish the following truth table. Is the last expression a tautology, contradiction or neither?

P	Q	$\sim Q$	$P \Rightarrow Q$	$P \lor \sim Q$	$(P \Rightarrow Q) \land (P \lor \sim Q)$
Т	T				
	F				
F	Т				
 F	F				

(2) Suppose that P is false and Q is true. Find whether each of these statements is true (T) or false (F).

$$\bullet \ (P \Rightarrow \sim Q) \Rightarrow Q$$

•
$$(P \land (Q \Longleftrightarrow (\sim P))) \lor Q$$

 \bullet Repeat the above problems with the alternate given information that P is false and Q is false.

$$\bullet \ (P \Rightarrow \sim Q) \Rightarrow Q$$

•
$$(P \land (Q \Longleftrightarrow (\sim P))) \lor Q$$

(3) Given the statement of implication " $(x \in \mathcal{S} \text{ and } x \leq 5)$ implies that (x > 2 or x = -10.)"

• Find its converse; write it without the word "not" and without the symbol "~."

• Find its negation; write it without the word "not" and without the symbol "~."

• Find its contrapositive; write it without the word "not" and without the symbol "~."

• Find its inverse; write it without the word "not" and without the symbol "~."

• If $S = \{3, 4, 7, 11\}$, is the statement true or false for all $x \in S$?

(4)	Given the statement: $\forall x \in \mathbb{Z}, (x \text{ even or } x 18) \Rightarrow ((x+1) \text{ is odd and } x^2 > 3).$ • Find its negation; write it without the symbol " \sim ."
	• Find a counterexample which proves the original statement is false.
(5)	Given the statement: $\forall x \in \mathbb{R}, \exists y \in \mathbb{Z} \text{ s.t. } yx \leq (yx + x).$ • Find its negation; write it without the symbol "~."
(6)	Given the statement: If you have a french-apple pie then you have raisins, cherries and a glazed crust. • Find its contrapositive; write it without the symbol "~."
	• Find its converse; write it without the symbol "~."
	• Rewrite the statement using the words "only if."
	• Rewrite the statement using the word "necessary."
	• Rewrite the statement using the word "sufficient."

- (7) Given universe $\mathcal{U} = \{1, 2, 3, 4, 5, 7, 9, 10, 21, 25\}$; $A = \{7, 9, 10, 21, 25\}$; and $B = \{5, 4, 7, 10, 21\}$. Find the following:
 - $\bullet \ \overline{A \cup \overline{B}}$
 - $\bullet (A B) \cup (B A)$
 - $\bullet \ \overline{\overline{(B-A)} \cap A}$
 - $|\mathcal{P}(A)|$
 - $|\mathcal{P}(A \times B) \times A|$
 - $\bullet \ A \cap \overline{A}$
 - $\mathcal{U} \overline{B}$
- (8) Given $A = \{4, \{5, 7\}, 7, \{7\}, \{\{5\}, 7\}\}.$
 - Find |A|

True or False?

- $\{\{5\}\}\in A$.
- $\{5\} \in A$.
- $5 \subseteq A$.
- $5 \in A$.
- $7 \in A$.
- $\{5,7\} \in A$.
- $\{7\} \in A$.
- $\{7\} \subseteq A$.
- $\bullet \ \{\{7\},7\} \subseteq A.$
- $\bullet \ \{\} \in A.$
- $\{\} \subseteq A$.