

Prove:  $\forall x \in S, P \Rightarrow Q.$

- Direct: Assume  $P$ , show  $Q$   $\square$
- Contrapositive: Assume  $\sim Q$ , show  $\sim P$ .  $\square$
- Contradiction: Assume  $P \wedge \sim Q$ , show (given) falsehood
- Induction: Show base:  $Q$  for smallest  $n$  in  $P$   $\square$   
Assume:  $Q$  for  $n = k$   $\square$   
Show:  $Q$  for  $n = k + 1$   $\square$

Translate Fact about division  $\rightsquigarrow$  Algebraic Equation

- $z$  even  $z = 2k$   $\square$
- $z$  odd  $z = 2m + 1$   $\square$
- $a \mid b$   $b = ap$   $\square$
- $z \bmod a = b$   $z = aq + b$   $\square$
- $a \equiv b \pmod{c}$   $a - b = cq$   $\square$

$\rightarrow$  Example:

$$(z^2 + 1) \equiv 17 \pmod{12} \rightsquigarrow z^2 + 1 - 17 = 12m$$

- Encrypt
- Find sequence  $a_n$
  - for A-Z letters, add  $a_n$  to standard number then  $\bmod 26$ , find new letter  $\square$
  - for binary, add  $a_n$  to bit, then  $\bmod 2$ .  $\square$

- Decrypt
- for A-Z, subtract  $a_n$  from std. number,  $\bmod 26$   $\square$
  - for binary, add  $a_n$  to bit,  $\bmod 2$ . (OR subtract)

Sets Given  $U, A, B$

- Find:  $\bar{A}, A - B, A \cup B, A \cap B, \mathcal{P}(A), A \times B, A \subseteq B?$   $\square$

$\rightarrow$  Example:

$$\text{Find } \overline{A \cup (B - A)} \text{ using DeMorgans}$$

- Find  $|A|, |A \cup B|, |A - B|, |\mathcal{P}(A)|, |A \times B|$   $\square$

$\rightarrow$  Example:

$$\text{Find } |\mathcal{P}(A \cup B) \times (A - B)|$$

Counting • Pizza orders, PIN numbers, DNA: Mult. Add. Subtract

- $n P_k, |A \cup B| = |A| + |B| - |A \cap B|$   $\square$

- $|\text{legal}| = |U| - |A| - |B| + |A \cap B|$   $\square$