

Ex: How many ways can we plan a bookcase, where we have 8 books, 4 shelves, and our plan only says how many books we plan to put on each shelf?

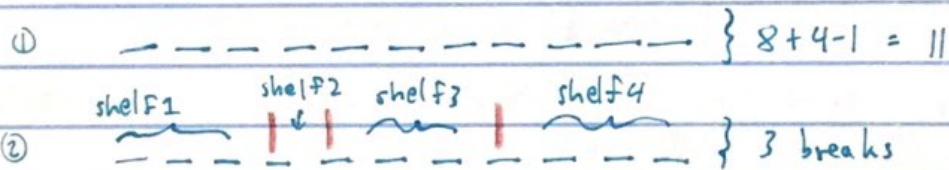


n books
k shelves

Answer: We have 8 identical "future book locations" ("—") to distribute among 4 distinct shelves.

- ① Add to the 8 blanks ("—") three more to account for shelf enders, or breaks ($3 = 4 - 1$)
- ② choose where to put the breaks, then see the remaining blanks as split up into shelves.

$\binom{n+k-1}{k-1}$
plans



→ 3 breaks are identical.

So the number of plans is 11 choose 3; $\binom{8+4-1}{4-1} = 165$

In general, n books on k shelves: Number of plans (giving # books planned for each shelf) is $\binom{n+k-1}{k-1}$.

Ex: How many ways are there to put 8 books on 4 shelves in ordered rows?

$\binom{n+k-1}{k-1} n!$

$= (n+k-1) P_n$

ways to
shelve.

Answer: First make the plan, then choose a book for every location in the plan.

$$\binom{8+4-1}{4-1} 8! = \binom{11}{3} 8! = \frac{11!}{3!8!} (8!) = \frac{11!}{3!} = 11 P_8 = 6,652,800$$