Notation : for whole number n

$$n! = n(n-1)(n-2)\cdots 2\cdot 1$$

$$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$$

$$1! = 1$$

by def: $0! = 1$

$$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 720$$

Notice : $n! = n\,(n-1)!$

$$n! = n P_n$$

$7!$ = number of 7-digit PINs with no repeating digits using only digits $1, 2, \ldots, 7$.

Division principle.

Idea: [count the number of structures (PINs, pizzas, ice cream cones, olympic podiums]

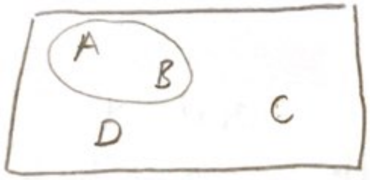
→ Sometimes, when using the multiplication principle (decisions, options) we count the number of ways to construct something but we really only want to count the final construction.

→ If each construction has the same number of ways, m, to construct it, then we can count them all, but then divide by m.

Ex: How many ways can you choose a committee of 2 people out of 4?

Construct: $\underset{4}{\_\_}\ \underset{3}{\_\_}\ \big\}\ 4\cdot 3 = 12$ | But each construction can be done 2 ways: Answer $\frac{12}{2} = \boxed{6}$

Picture



People $\{A, B, C, D\}$

Committees:

$\{A, B\}$ same as $\{B, A\}$
$\{A, C\}$ same as $\{C, A\}$
$\{A, D\}$ same as $\{D, A\}$
$\{B, C\}$ same as $\{C, B\}$
$\{B, D\}$ same as $\{D, B\}$
$\{C, D\}$ same as $\{D, C\}$

Ex: How many subsets of size 2 are there of $U = \{1, \dots, 10\}$?

Ans: $\frac{10 \cdot 9}{2} = \boxed{45}$

Ex: How many subsets of size 3 are there of $U = \{1, \dots, 10\}$?

Ans $\frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$

$\boxed{= 120}$

For a given subset $\{3, 9, 2\}$
We can build it $3 \cdot 2 \cdot 1$ ways $\rightarrow$ those are the orders of picking the three numbers

3, 9, 2
3 2 9
9 2 3
9 3 2
2 3 9
2 9 3

Notation: the number of subsets of size m from a set of size n (like $\{1, \dots, n\}$)

is $\boxed{\dfrac{n(n-1)(n-2)\cdots(n-m+1)}{m(m-1)(m-2)\cdots 1} = \dfrac{nP_m}{m!}}$

Def: The subsets of size m are called $\boxed{\text{m-combinations.}}$

Short hand: Def: $\boxed{\dfrac{nP_m}{n!} = \dbinom{n}{m} = nC_m = \text{"n choose m"}}$

Handy: multiply on top and bottom by $(n-m)!$ to get $\dfrac{nP_m\,(n-m)!}{m!\,(n-m)!} = \dfrac{n!}{m!\,(n-m)!}$

Summary: For $U = \{1, 2, \dots, n\}$; Then:

The number of subsets of size m

= the number of m-combinations of n

= n choose m

$= nC_m$

$= \dbinom{n}{m}$

$= \dfrac{n(n-1)\cdots(n-m+1)}{m(m-1)\cdots 1}$

$= \dfrac{nP_m}{m!}$

$= \dfrac{n!}{m!\,(n-m)!}$