and $|\overline{A \cup B \cup C}| = |U| - |A| - |B| - |C| + |A \cap B| + |A \cap C| + |B \cap C| - |A \cap B \cap C|$

$= 7 - 6 = 1$

Example

How many PIN's with 5 digits

but: • no repeated digits
• First digit cannot be 0
• Third digit cannot be 2
• Fifth digit cannot be 5

Idea: let U be all the 5 digit PINS with no repeated digits

$|U| = 10P_5 = 30{,}240$

let A be PIN's with first digit 0.
let B be PINs with third digit 2.
let C be PINs with fifth digit 5.

Then legal PINs are $\overline{A \cup B \cup C}$.

$|A| = |B| = |C| = 3024$
$|A \cap B| = |B \cap C| = |A \cap C| = 336$
$|A \cap B \cap C| = 42$

$|\overline{A \cup B \cup C}| = 30240 - 3(3024) + 3(336) - 42$

$= \boxed{22{,}134}$

Example: Count 5-digit PINs with no repeated digits, and either start with 7 or end with 8?

let A = start with 7, B = end with 8.

$|A| + |B| - |A \cap B|$

$= 3024 + 3024 - 336 = \boxed{5712}$