Problems: board

$U = \{1, \ldots, 9\}$; $A = \{3,5,8\}$, $B = \{2,3,5,7\}$

Find: $A \cup B = \{2,3,5,7,8\}$

$A \cap B = \{3,5\}$

$A^c \cup B = \bar{A} \cup B = \{2,3,5,7,1,4,6,9\}$

Always true!
$B - A = \bar{A} \cap B = \{2,7\}$

$\overline{A \cap B} = \bar{A} \cup \bar{B} = \{1,2,4,6,7,8,9\}$

$\overline{A \cup (\overline{\bar{B} \cap A})}$	$= \overline{A \cup (B \cup \bar{A})}$	$= \overline{A \cup B \cup \bar{A}}$
$= \overline{A \cup \overline{\{8\}}}$	$= \bar{A} \cap (\overline{B \cup \bar{A}})$	$= \overline{U}$
$= \overline{A \cup \{1,2,3,4,5,6,7,9\}}$	$= \bar{A} \cap (\bar{B} \cap A)$	$= \phi$
$= \overline{\{1,2,3,4,5,6,7,8,9\}}$	$= \bar{A} \cap \bar{B} \cap A$	
$= \{\} = \phi$		
	$= \phi$	

We used:
① $\overline{U} = \phi$, $A \cap \bar{A} = \phi$

② $A \cup \bar{A} = U$

③ De Morgan $\overline{A \cap B} = \bar{A} \cup \bar{B}$

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

④ $\bar{\bar{A}} = A$

⑤ $A \cap (B \cap C) = (A \cap B) \cap C$, $A \cup (B \cup C) = (A \cup B) \cup C$

Notation

$$\bigcup_{i=1}^{3} \{i, i+10\} = \{1,11\} \cup \{2,12\} \cup \{3,13\}$$
$$= \{1, 2, 3, 11, 12, 13\}$$

Cartesian Product : $A = \{4, 5, 7\}$, $B = \{9, 5\}$

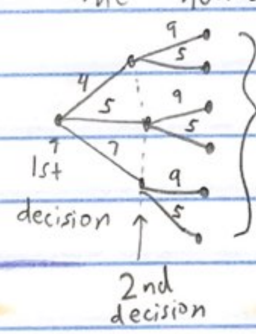
$A \times B$ = the set of ordered pairs (a, b), $a \in A$, $b \in B$

$= \{ (a,b) \mid a \in A, b \in B\}$

$= \{(4,9), (4,5), (5,9), (5,5), (7,9), (7,5)\}$

$|A \times B| = 6 = 3 \cdot 2 = |A| \cdot |B|$ ... always!

Proof: the number of pairs equals the number of first options times the number of second options.



total possibilities.

Counting Principles

Multiplication: If you have several decisions to make, and the number of options for each decision is the same no matter what the earlier decisions were (independent), then the total number of possibilities is the multiplication (product) of the numbers of options for the separate decisions