

(1) Finish the following truth table.

P	Q	$\sim Q$	$P \Rightarrow Q$	$P \vee \sim Q$	$(P \Rightarrow Q) \wedge (P \vee \sim Q)$
T	T	F	T	T	T
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

(2) Suppose that P is false and Q is true. Find whether each of these statements is true (T) or false (F).

• $(P \Rightarrow \sim Q) \Rightarrow Q$
T

• $(P \wedge (Q \iff (\sim P))) \vee Q$
T

• Repeat the above problems with the alternate given information that $Q \Rightarrow P$ is false.
T, T

(3) Given the statement of implication “ $(x \in \mathcal{S} \text{ and } x \leq 5)$ implies that $(x > 2 \text{ or } x = -10)$.”

• Find its converse; write it without the symbol “ \sim .”
 $(x > 2 \text{ or } x = -10)$ implies that $(x \in \mathcal{S} \text{ and } x \leq 5)$

• Find its negation; write it without the symbol “ \sim .”
 $(x \in \mathcal{S} \text{ and } x \leq 5)$ and $(x \leq 2 \text{ and } x \neq -10)$.

• Find its contrapositive; write it without the symbol “ \sim .”
 $(x \leq 2 \text{ and } x \neq -10) \Rightarrow (x \notin \mathcal{S} \text{ or } x > 5)$

• Find its inverse; write it without the symbol “ \sim .”
 $(x \notin \mathcal{S} \text{ or } x > 5) \Rightarrow (x \leq 2 \text{ and } x \neq -10)$

• If $\mathcal{S} = \{3, 4, 7, 11\}$, is the original statement true or false for all x ?
T

(4) Given the statement: $\forall x \in \mathbb{Z}, (x \text{ even or } x|18) \Rightarrow ((x+1) \text{ is odd and } x^2 > 3)$.

• Find its negation; write it without the symbol “ \sim .”
 $\exists x \in \mathbb{Z}$ such that $(x \text{ even or } x|18)$ and $((x+1) \text{ is even or } x^2 \leq 3)$.

• Find a counterexample which proves the original statement is false.
 $x = 9$. Also good answers: $x = 0, 1, 3$

² (5) Given the statement: $\forall x \in \mathbb{R}, \exists y \in \mathbb{N}$ s.t. $yx \leq (yx + x)$.

- Find its negation; write it without the symbol “ \sim .”
 $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{N}, yx > (yx + x)$.

(6) Given the statement: If you have a french-apple pie then you have raisins, cherries and a glazed crust.

- Find its contrapositive; write it without the symbol “ \sim .”
If you don't have (raisins, cherries and a glazed crust) then you don't have a french-apple pie.

- Find its converse; write it without the symbol “ \sim .”
If you have raisins, cherries and a glazed crust then you have a french-apple pie. (Note: not true!)

- Rewrite the original statement using the words “only if.”
You have a french-apple pie only if you have raisins, cherries and a glazed crust.

- Rewrite the statement using the word “necessary.”
Having raisins, cherries and a glazed crust is necessary for having a french-apple pie.

- Rewrite the statement using the word “sufficient.”
Having a french-apple pie is sufficient for having raisins, cherries and a glazed crust.

(7) All quiz problems are good to study from! Homework problems too—especially go over some more proofs from chapter 3.