**Combinatorics. Review for Test 2**     Don't simplify the generating functions you find: after getting the closed form just leave them as found.

1.a) Given an e.g.f. $f(x) = 3e^x + 2x$, for a sequence $a_n$, find a closed formula for $a_n$.

b) Given an e.g.f. $f(x) = e^{3x} + 4$, for a sequence $a_n$, find a closed formula for $a_n$.

2.a) Find the o.g.f. for the number of ways to choose $n$ donuts from a menu that offers two types: chocolate and plain. You must choose exactly 4 or 5 chocolates, and a nonzero even number of plain.

b) Find the o.g.f. for the number of ways to choose $n$ donuts from a menu that offers two types: chocolate and plain, where you must choose at least 2 chocolate and an odd number greater than 3 of plain.

c) Given an o.g.f. for a sequence $a_n$: $f(x) = \dfrac{5}{(1-x)^2} + 3 + 7x^2$, find a closed formula for $a_n$.

3.a) Find the e.g.f. for the number of ways to arrange a permutation of length $n$ using the letters $A, C$, with repetition, where there are an odd number of $A$'s, and exactly 3 or 7 $C$'s.

b) Find the e.g.f. for the number of ways to arrange a permutation of length $n$ using the letters $A, C, T$, with repetition, where there are at least 2 $A$'s, an even number of $C$'s, and any number of $T$'s.

4. Given the recurrence relation for a sequence $a_n = 3a_{n-3} + 7$; $n \geq 3$; $a_0 = 3$, $a_1 = 0$, $a_2 = 5$.

(a) Find $a_3, a_4$, and $a_5$.

(b) Find the o.g.f. $f(x)$ for $a_n$.

5. For the graphs A,B,C pictured, answer the following:

A

B

C
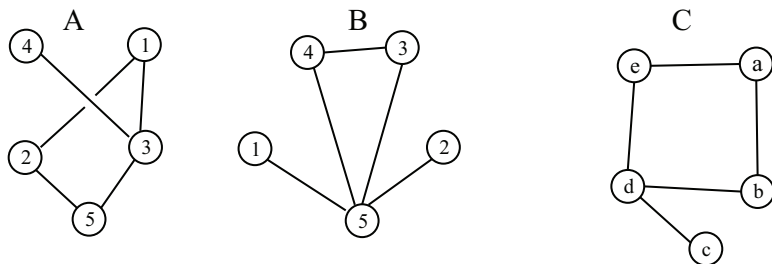
a) Find the diameters:

diam(A)=

diam(B)=

b) Find the degree sequence deg.seq.(B)

c) Write in 'yes' or 'no.' In A, is 1,3,5,2,1,2,1

...a cycle?_____ ...is it a trail?_____ ...is it a walk?_____

d) Write in 'yes' or 'no.' In B, is 1,3,5,2,1

...a walk?_____ ...is it a trail?_____ ...is it a cycle?_____

e) Find an isomorphism $f$ from A to C. (List the inputs and outputs $f(\_) = \_$ for your isomorphism.)

6. Find the number of permutations $\varphi$ of $\{1, 2, 3, 4, 5\}$ where $\varphi(1) \neq 2, 5$, $\varphi(4) \neq 1, 3, 5$, and $\varphi(5) \neq 2, 3$.

7.a) Use the o.g.f. for $a_n$ which is $f(x) = \frac{x}{1-3x}$ to find the value of $a_2$.

b) Use the e.g.f. for $a_n$ which is $f(x) = xe^{2x}$ to find the value of $a_1$.