Combinatorics. Quiz 4. Name

1. Find the number of non-negative integer solutions to the equation  $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ where  $x_2 \le 7$  and  $x_3 \le 12$ .

2. Find the number of ways to purchase 20 donuts if the shop has 4 types: chocolate, krueller, glazed and plain, where: 1) you must purchase at least 2 chocolates but no more than 10 chocolates, and 2) also you must purchase no more than 15 glazed and no more than 17 plain.

3. Find the number of solutions to the integer equation  $x_1 + x_2 + x_3 = 15$  where  $0 \le x_1 \le 4$ ,  $0 \le x_2 \le 5$  and  $0 \le x_3 \le 7$ . Give your answer as a whole number.

4. Consider a grid graph with  $10 \times 7$  edges. Find the number of shortest walks from the lower left corner (0,0) to the upper right (10,7), where the walk may not pass through any of the following nodes: (0,3), (4,5), (6,4) or (7,7).