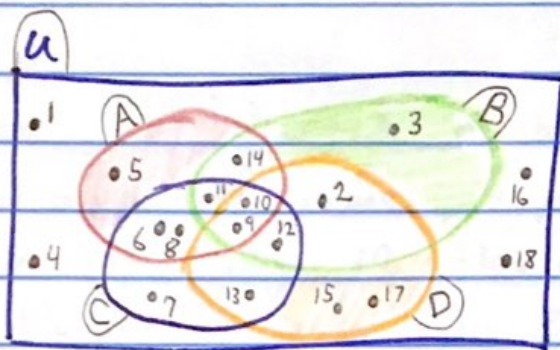


More examples with inclusion-exclusion



$$U = \{1, \dots, 18\} = [18]$$

How many elements are in U but not in A , not in B , not in C , and not in D ?
Find $|\overline{A \cup B \cup C \cup D}|$.

If you can, just count them! $\overline{A \cup B \cup C \cup D}$ from the picture, equals $\{1, 4, 16, 18\}$ so the answer is $\boxed{4}$.

If there are too many to count:

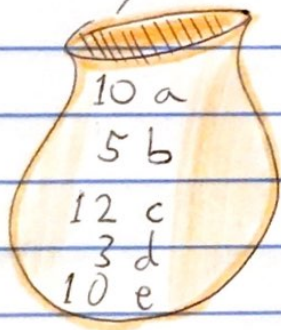
$$\begin{aligned}
 |\overline{A \cup B \cup C \cup D}| &= |U| - |A| - |B| - |C| - |D| \\
 &\quad + |A \cap B| + |B \cap C| + |A \cap D| + |A \cap C| + |B \cap D| + |C \cap D| \\
 &\quad - |A \cap B \cap C| - |A \cap C \cap D| - |A \cap B \cap D| - |B \cap C \cap D| \\
 &\quad + |A \cap B \cap C \cap D|
 \end{aligned}$$

$$\begin{aligned}
 &= 18 - 7 - 7 - 8 - 7 \\
 &\quad + 4 + 4 + 2 + 5 + 4 + 4 \\
 &\quad - 3 - 2 - 2 - 3 + 2 = \boxed{4}
 \end{aligned}$$

Example:

We are purchasing a bag of 40 donuts.
The shop has only 5 flavors; a, b, c, d, e.
There are limited numbers available of some types: the shop has only 12 flavor a, only 5 flavor b, and only 20 flavor d.
How many possible 40 donut bags are there?

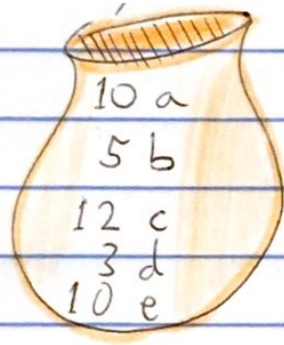
Ex:



... but not allowed:

13 a, 6 b, or 21 d.
($a \geq 13$) ($b \geq 6$) ($d \geq 21$)

Ex:



... but not allowed:

13 a, 6 b, or 21 d.
 $(a \geq 13)$ $(b \geq 6)$ $(d \geq 21)$

Answer: Let U = all theoretical bags if all types are unlimited.

$$|U| = \binom{40+5-1}{5-1}$$

So: we use inclusion - exclusion.

$\binom{40+5-1}{5-1} - \binom{27+5-1}{5-1} - \binom{34+5-1}{5-1} - \binom{19+5-1}{5-1}$ <p style="text-align: center;"> \swarrow $\binom{40-13}{}$ \swarrow $\binom{40-6}{}$ \swarrow $\binom{40-21}{}$ $a \geq 13$ $b \geq 6$ $d \geq 21$ </p>	$= \binom{44}{4} - \binom{31}{4} - \binom{38}{4} - \binom{23}{4}$
$+ \binom{21+5-1}{5-1} + \binom{6+5-1}{5-1} + \binom{13+5-1}{5-1}$ <p style="text-align: center;"> \swarrow $\binom{40-19}{}$ \swarrow $\binom{40-34}{}$ \swarrow $\binom{40-27}{}$ $a \geq 13, b \geq 6$ $a \geq 13, d \geq 21$ $b \geq 6, d \geq 21$ </p>	$+ \binom{25}{4} + \binom{10}{4} + \binom{17}{4}$
$- \binom{0+5-1}{5-1}$ <p style="text-align: center;"> \swarrow $\binom{40-40}{}$ $a \geq 13, b \geq 6, d \geq 21$ </p>	$- \binom{4}{4}$
<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p>Note: sometimes later terms will be 0; like $\binom{2}{4} = 0$</p> </div>	$= 135751 - 31465 - 73815 - 8855 + 12650 + 210 + 2380 - 1$
	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> $= 36855$ </div>