Ex: a_3 :

0 u's				2 u's		
CCC	CCA	CAC	ACC	UUC	UCU	CUU
AAA	AAC	ACA	CAA	UUA	UAU	AUU

Let $g(x)$ = the E.g.f. for a_n the number of RNA sequences length n using U, C, A with either 0 or 2 U's (any number of C's, A's).

$$g(x) = \left(1 + \frac{x^2}{2!}\right) e^x e^x$$

$$= e^{2x} + \frac{1}{2} x^2 e^{2x}$$

→ Find a closed form for a_n.

$$g(x) = \sum_{n=0}^{\infty} 2^n \frac{x^n}{n!} + \frac{1}{2} \sum_{n=0}^{\infty} 2^n x^2 \frac{x^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{2^n x^n}{n!} + \frac{1}{2} \sum_{n=0}^{\infty} 2^n \frac{x^{n+2}}{n!}$$

$n \to n-2$

$$+ \frac{1}{2} \sum_{\substack{n=0 \\ -2 \; +2}}^{\infty} \frac{2^{n-2} x^n}{(n-2)!}$$

$$+ \frac{1}{2} \sum_{n=2}^{\infty} \frac{2^{n-2} n(n-1) x^n}{n!}$$

$n=0, 1$ give 0

$$\Rightarrow \boxed{a_n = 2^n + \frac{1}{2} 2^{n-2} n(n-1)}$$

$a_4 = 16 + \frac{1}{2}(4)(4)(3)$

$= 40$

$a_3 = 2^3 + \frac{1}{2} 2(3)(2)$

$= \boxed{14}$ check at top.

Example

Find the e.g.f. for the number q_n of RNA sequences (length n) using U, C, G (no adenine) which have at least 2 "U"s, no more than 4 "C"s, at least three "C"s and an odd number of "G"s.

$$g(x) = \left(\frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)\left(\frac{x^3}{3!} + \frac{x^4}{4!}\right)\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)$$

$$= \boxed{\left(e^x - x - 1\right)x^3\left(\frac{1}{6} + \frac{x}{24}\right)\left(\frac{e^x - e^{-x}}{2}\right)}$$

$$= \left(\frac{x^3}{6} + \frac{x^4}{24}\right)\left(\frac{1}{2}\right)\left(e^{2x} - 1 - xe^x + xe^{-x} - e^x + e^{-x}\right)$$

Ex. Find the e.g.f for the number p_n of RNA sequences (length n) using U, C, A (no guanine) with 2 or 3 U's only and at least 2 C's.

$$g(x) = \left(\frac{x^2}{2} + \frac{x^3}{3!}\right)\left(\frac{x^2}{2} + \frac{x^3}{3!} + \cdots\right)\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots\right)$$

$$= \left(\frac{x^2}{2} + \frac{x^3}{3!}\right)\left(e^x - \frac{x}{1!} - 1\right)e^x$$

$$= \left(\frac{x^2}{2} + \frac{x^3}{6}\right)\left(e^{2x} - xe^x - e^x\right)$$