Permutations

Recall "7 permute 3"

$= {}_7P_3$

$= 7 \cdot 6 \cdot 5 \quad = 7!/4!$

= # of 3-digit, ordered lists (PINs) using 1-7 $\{1, \dots, 7\}$ without repeating digits

Now "7 permute 7"

$= 7!/0!$

$= 7!$

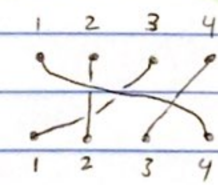
= # ordered lists of $\{1, \dots, 7\}$ (use all 7 digits without repeating)

= # <u>the</u> permutations of $\{1, \dots, 7\}$

Very useful! Lots of ways to draw one permutation of $\{1, \dots, 4\}$.

Ex: $\boxed{\underline{4}\ \underline{2}\ \underline{1}\ \underline{3}}$

Ⓐ



string picture: inputs at top (positions in the list) filled in by outputs at bottom.

Ⓑ list 4,2,1,3

outputs $i_1 i_2 i_3 i_4$ (i)

Call this permutation φ

Ⓒ table:

→ function

→ 1-1

input j	output $\varphi(j)$
1	$4 = \varphi(1)$
2	$2 = \varphi(2)$
3	$1 = \varphi(3)$
4	$3 = \varphi(4)$

Ⓓ grid "chess board"

← outputs

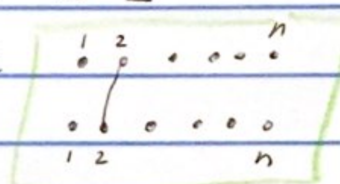
input

φ	1	2	3	4
1				1
2		1		
3	1			
4			1	

only one 1 in each row & column.

Counting "special" permutations (extra requirements)

→ The number of permutations of
$\{1, \dots, n\} = [n]$ is $n!$

→ The number of permutations of
$\{1, \dots, n\}$ such that $\varphi(2) = 2$
... same as "second digit is 2"
or in picture, require 

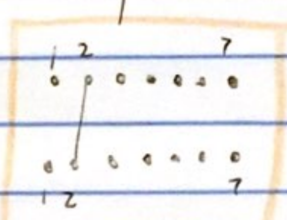
$\Rightarrow (n-1)\,1\,(n-2)(n-3)\cdots 1$
$= \boxed{(n-1)!}$

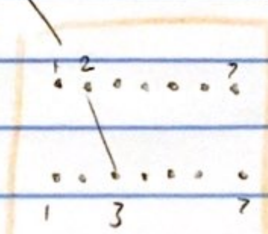
→ The number of permutations of $[n]$
such that $\varphi(2) \neq 2$
... same as "second digit is not 2"
$\boxed{n! - (n-1)!}$

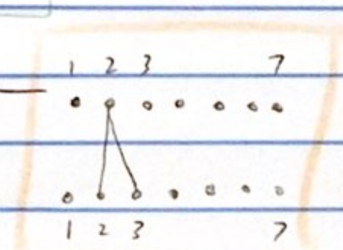
→ The number of permutations of $[7]$
such that $\varphi(2) \neq 2$ and $\varphi(2) \neq 3$

$\boxed{7! - 6! - 6! + 0} = \boxed{3600}$

total







illegal: no perms
with $\varphi(2) = 2$ & 3