
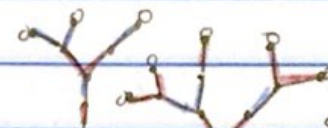
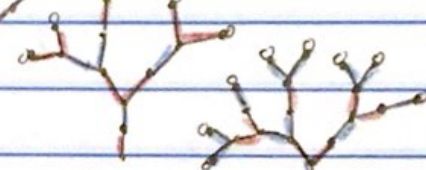
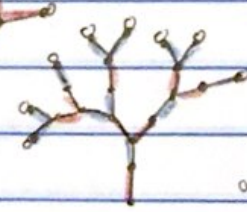



Ex: $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$; $a_0 = 1, a_1 = 1$

n	a_n	Fibonacci
0	1	1
1	1	1
2	2	
3	3	
4	5	
5	8	
6	13	

$$\sum_{n=2}^{\infty} a_n x^n = \sum_{n=2}^{\infty} a_{n-1} x^n + \sum_{n=2}^{\infty} a_{n-2} x^n$$

Goal: { use algebra to get $\sum_{n=0}^{\infty} a_n x^n = f$ }
 (also some recognizable Maclaurin series) }
 (Note: 'zero' and 'Match' are circled in the original image with arrows pointing to the $n=0$ term and the right-hand side of the equation respectively.)

Methods: 1) add & subtract, to a sum

$$\sum_{n=0}^{\infty} a_n x^n - a_1 x - a_0 = \sum_{n=2}^{\infty} a_{n-1} x^n + \sum_{n=2}^{\infty} a_{n-2} x^n$$

2) Multiply & divide by powers of x , to a sum

$$\sum_{n=0}^{\infty} a_n x^n - 1x - 1 = x \sum_{n=2}^{\infty} a_{n-1} x^{n-1} + x^2 \sum_{n=2}^{\infty} a_{n-2} x^{n-2}$$

3) Reindex

$$f - x - 1 = x \sum_{n=1}^{\infty} a_n x^n + x^2 \sum_{n=0}^{\infty} a_n x^n$$

More!

$$\Rightarrow f - x - 1 = x \left(\sum_{n=0}^{\infty} a_n x^n - a_0 x^0 \right) + x^2 f$$

$$\Rightarrow f - x - 1 = x(f - 1) + x^2 f$$

Solve for f :

$$\Rightarrow f - x - 1 - xf - x^2 f = -x$$

$$\Rightarrow f - xf - x^2 f = 1$$

$$f(1 - x - x^2) = 1$$

$$f = \frac{1}{1 - x - x^2}$$

$$= 1 + x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + 13x^6 + \dots$$