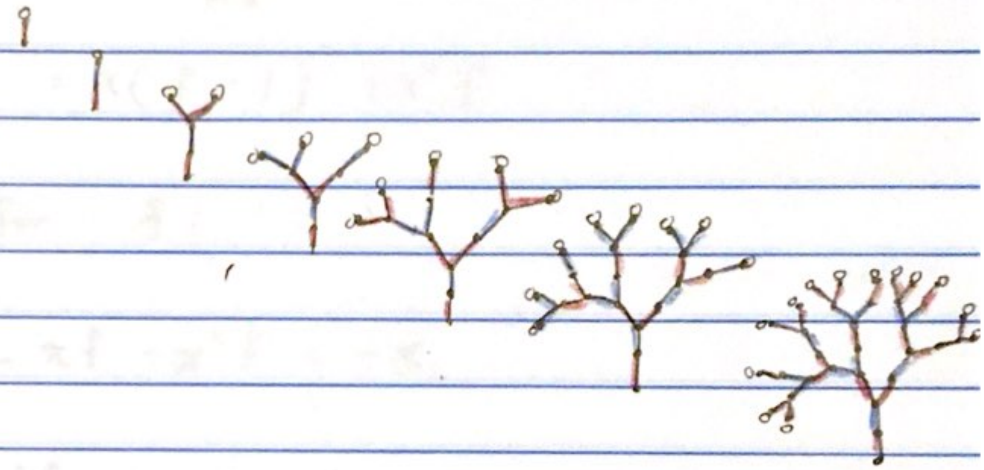
Ex: $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$; $a_0 = 1, a_1 = 1$

n	a_n
0	1
1	1
2	2
3	3
4	5
5	8
6	13

Fibonacci



$$\sum_{n=2}^{\infty} a_n x^n = \sum_{n=2}^{\infty} a_{n-1} x^n + \sum_{n=2}^{\infty} a_{n-2} x^n$$

Goal: $\left\{ \begin{array}{l} \text{use algebra to get } \sum_{n=0}^{\infty} a_n x^n = f \\ \text{(also some recognizable Maclaurin series)} \end{array} \right\}$

zero

Match

Methods: 1) add + subtract, to a sum

$$\sum_{n=0}^{\infty} a_n x^n - a_1 x - a_0 = \sum_{n=2}^{\infty} a_{n-1} x^n + \sum_{n=2}^{\infty} a_{n-2} x^n$$

2) Multiply & divide by powers of x, to a sum

$$\sum_{n=0}^{\infty} a_n x^n - \underline{1}x - 1 = x \sum_{n=2}^{\infty} a_{n-1} x^{n-1} + x^2 \sum_{n=2}^{\infty} a_{n-2} x^{n-2}$$

3) Reindex

$$f - x - 1 = x \sum_{n=1}^{\infty} a_n x^n + x^2 \sum_{n=0}^{\infty} a_n x^n$$

More!

$$\Rightarrow \quad f - x - 1 = x\left(\sum_{n=0}^{\infty} a_n x^n - a_0 x^0\right) + x^2 f$$

$$\Rightarrow \quad f - x - 1 = x(f - 1) + x^2 f$$

Solve for f:

$$\Rightarrow \quad f - x - 1 - xf - x^2 f = -x$$

$$\Rightarrow \quad f - xf - x^2 f = 1$$

$$f(1 - x - x^2) = 1$$

$$\boxed{f = \frac{1}{1 - x - x^2}}$$

$$= 1 + x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + 13x^6 + \cdots$$