

## Combinatorics Test 1 Fall'20 Review answers

1. Given universe  $U = \{1, 2, 3, \dots, 107\}$ ;  $A = \{7, 9, 10, 21, 25\}$ ; and  $B = \{5, 4, 7, 10, 21\}$ . Find the following:

•  $|P(A)|$

$$2^5 = 32$$

• The number of subsets of  $B$  of size 3.

$$\binom{5}{3} = 10$$

•  $|A \cup B|$

$$7$$

•  $|\overline{A \cup B}|$

$$100$$

2. How many PIN's are there with 7 digits, no repeated digits?

$$10P_7 = 604,800$$

3. How many PIN's are there with 4 digits, no repeated digits, and such that they obey the rule that: either the third digit is 0, the second digit is 2, or the last digit is 1? (more than one requirement can also be true.)

$$3(504) - 3(56) + 7 = 1351$$

4. How many PIN's are there with 3 digits, repeated digits allowed, and such that the first digit is not 0 and the second digit is not 9?

$$1000 - 100 - 100 + 10 = 810$$

5. How many ways can 7 students fill in the first row of 4 seats? (seated in order, leaving 3 students still standing.)

$$7P_4 = 840$$

6. How many different committees of 4 people can be selected from a group of 10 people?

$$\binom{10}{4} = 210$$

7. How many ways can 3 books be distributed to 7 shelves on a bookcase? (No ordering of the books on the shelves, just a loose pile.)

$$7^3 = 343$$

8. How many ways can we plan for 3 books to be placed on a bookcase with 7 shelves? (No books on the shelves yet, just the plan.)

$$\binom{3+7-1}{7-1} = 84$$

9. How many ways are there to put 3 books on the 7 shelves of the bookcase in ordered rows?

$$\binom{3+7-1}{7-1} 3! = 504$$

10. How many ways can we plan for 3 books to be placed on a bookcase with 7 shelves if at least one book must go on the top shelf? (No books yet, just the plan.)

$$1 \cdot \binom{2+7-1}{7-1} = \binom{8}{6} = 28$$

11. How many ways are there to put 3 books on the 7 shelves of the bookcase in ordered rows if at least one book must go on the top shelf?

$$\binom{2+7-1}{7-1} 3! = 168$$

12. How many ways can 7 books be distributed to 3 shelves on a bookcase? (No ordering of the books on the shelves, just a loose pile.)

$$3^7 = 2187$$

13. How many ways can we plan for 7 books to be placed on a bookcase with 3 shelves? (No books on the shelves yet, just the plan.)

$$\binom{7+3-1}{3-1} = \binom{9}{2} = 36$$

14. How many ways are there to put 7 books on the 3 shelves of the bookcase in ordered rows?

$$\binom{7+3-1}{3-1} 7! = 181,440$$

15. How many ways can we plan for 7 books to be placed on a bookcase with 3 shelves if at least two books must go on the top shelf? (No books yet, just the plan.)

$$\binom{5+3-1}{3-1} = \binom{7}{2} = 21$$

16. How many ways are there to put 7 books on the 3 shelves of the bookcase in ordered rows if at least two books must go on the top shelf?

$$\binom{5+3-1}{3-1} 7! = 105,840$$

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$$\binom{9}{3} - \binom{2}{1} \binom{7}{2} - \binom{6}{2} \binom{3}{1} + \binom{2}{1} \binom{4}{1} \binom{3}{1}$$

$$- \binom{4}{1} \binom{4}{0} - \binom{2}{1} \binom{1}{1} \binom{4}{0} = 2$$

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$$\binom{19}{4}$$

$$\binom{19}{4} - \binom{12+4}{4} - \binom{11+4}{4} - \binom{13+4}{4} + \binom{10+4}{4} + \binom{9+4}{4} + \binom{8+4}{4} - \binom{6+4}{4}$$

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a)  $\binom{26}{6}$

b)  $7^{20}$

c)  $\binom{26}{6} 20!$