Combinatorics Fall'20 Review 1

1. Given universe $\mathcal{U} = \{1, 2, 3, \ldots, 107\}$; $A = \{7, 9, 10, 21, 25\}$; and $B = \{5, 4, 7, 10, 21\}$. Find the following:
   - $|\mathcal{P}(A)|$

   - The number of subsets of B of size 3.

   - $|A \cup B|$

   - $|\overline{A \cup B}|$

2. How many PIN's are there with 7 digits, no repeated digits?

3. How many PIN's are there with 4 digits, no repeated digits, and such that they obey the rule that: either the third digit is 0, the second digit is 2, or the last digit is 1? (more than one requirement can also be true.)

4. How many PIN's are there with 3 digits, repeated digits allowed, and such that the first digit is not 0 and the second digit is not 9?

5. How many ways can 7 students fill in the first row of 4 seats? (seated in order, leaving 3 students still standing.)

6. How many different committees of 4 people can be selected from a group of 10 people?

7. How many ways can 3 books be distributed to 7 shelves on a bookcase? (No ordering of the books on the shelves, just a loose pile.)

8. How many ways can we plan for 3 books to be placed on a bookcase with 7 shelves? (No books on the shelves yet, just the plan.)

9. How many ways are there to put 3 books on the 7 shelves of the bookcase in ordered rows?

10. How many ways can we plan for 3 books to be placed on a bookcase with 7 shelves if at least one book must go on the top shelf? (No books yet, just the plan.)

11. How many ways are there to put 3 books on the 7 shelves of the bookcase in ordered rows if at least one book must go on the top shelf?

12. How many ways can 7 books be distributed to 3 shelves on a bookcase? (No ordering of the books on the shelves, just a loose pile.)

13. How many ways can we plan for 7 books to be placed on a bookcase with 3 shelves? (No books on the shelves yet, just the plan.)

14. How many ways are there to put 7 books on the 3 shelves of the bookcase in ordered rows?

15. How many ways can we plan for 7 books to be placed on a bookcase with 3 shelves if at least two books must go on the top shelf? (No books yet, just the plan.)

16. How many ways are there to put 7 books on the 3 shelves of the bookcase in ordered rows if at least two books must go on the top shelf?

17. On page 2, question 5 of quiz 3, we again are walking east and north from the lower left corner to the upper right of the neighborhood map of Parma: 3 blocks by 6 blocks.

   • How many ways to walk if you may not visit the corner of Gerald and 67th, and you may not visit the corner of Forest and 60th?

   • How many ways to walk if you may not use the block of Gerald from 67th to 60th, but you must use the block of 54th between Gerald and Kenneth?

18. You must distribute 20 identical donuts to five knights. (This is the same as buying 20 donuts of 5 types.)

   • How many ways to distribute them if each knight must get at least one donut?

   • How many ways to distribute them if each knight must get at least one donut, but Arthur can have no more than 3, Gawain can have no more than 4, and Lancelot can have no more than 2?

19. Do number 45 from chapter 2 (page 66), but with 20 books on 7 shelves.

20. Study all quiz questions and examples from notes!