Project for Combinatorics

Required steps to completion:

0) Due date: Oct 30. Choose an entry in the OEIS with at least two structures (graphical interpretations) and which allows the following outline to be filled in. Get approval from Dr. Forcey, by emailing the outline, as follows:
   a) The sequence ID, such as A000108
   b) The first 10 terms: 1, **1**, **2**, **5**, **14**, **42**, 132, 429, 1430, 4862
   c) The two interpretations you plan to illustrate.
      i) Number of ways to insert n pairs of parentheses in a word of n+1 letters. E.g., for n=2 there are 2 ways: ((ab)c) or (a(bc)); for n=3 there are 5 ways: ((ab)(cd)), (((ab)c)d), ((a(bc))d), (a((bc)d)), (a(b(cd))).
      ii) Consider all the binomial(2n,n) paths on squared paper that (i) start at (0, 0), (ii) end at (2n, 0) and (iii) at each step, either make a (+1,+1) step or a (+1,-1) step. Then the number of such paths that never go below the x-axis (Dyck paths) is C(n).
   d) The four formulas of at least three types: closed, recursive, G.f. and E.g.f.
      i) a(n) = binomial(2*n, n)-binomial(2*n, n-1).
      ii) a(n) = Sum_{k=0..n-1} a(k)a(n-1-k).
      iii) a(n) = Prod_{k=2..n} (1 + n/k), if n>1.
      iv) G.f.: A(x) = (1 - sqrt(1 - 4*x)) / (2*x). G.f. A(x) satisfies A = 1 + x*A^2.
      v) E.g.f.: exp(2*x)*(I_0(2*x)-I_1(2*x)), where I_n is Bessel function.

The remaining steps are due in two stages: Rough draft by Nov 30, Final draft by Dec. 7.

1) Illustrate both structures for the sets counted by a_n1, a_n2 where Dr. Forcey assigns n1,n2.
2) Four demonstrations, subject to the types of the four formulas from step 0.d).
   Demonstrate a closed formula for n1,n2, where Dr. Forcey assigns n1,n2.
   Demonstrate a recursion for j, j+1, j+2, where Dr. Forcey assigns j.
   Demonstrate the G.f. for n1,n2, where Dr. Forcey assigns n1,n2
   Demonstrate the E.g.f. for n1,n2, where Dr. Forcey assigns n1,n2.
3) Experimental bijection: find a rule for one of your structures that maps it to the other, for a set counted by a_n1. Show how the rule works on the next size: a_(n1+1).
4) Experimental superstructure: find a way to organize the set for one of the structures counted by a_n1. Show how the rule works on the next size: a_(n1+1). Show how the rule works on the other structure, via your bijection from step 3).
5) Open question: Describe an open question about your sequence…either by researching or by your own invention.