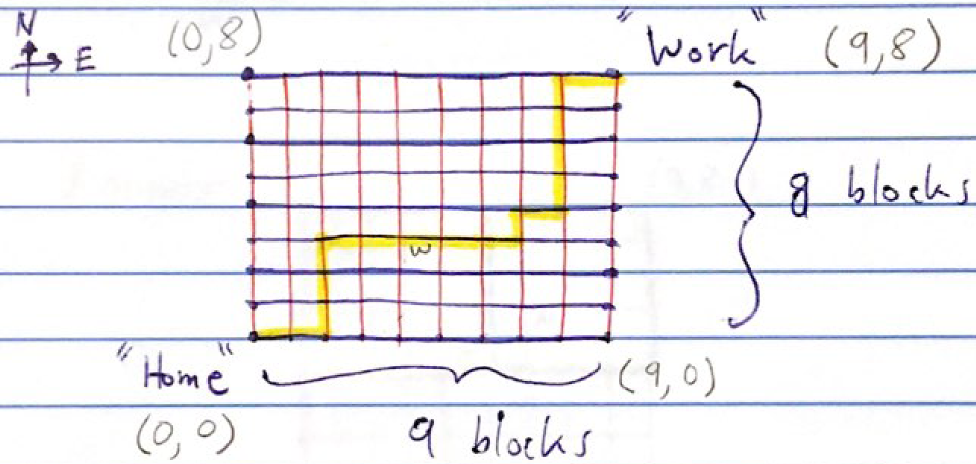
Another application of anagrams (multipermutations) is network routing.

Example from the book:



This is a grid network.

1) How many ways can we walk (route) from node (0,0) ("Home") to node (9,8) ("Work")? Only steps East and North are allowed. (positive x + y)

Ex: highlighted path = EENNNEEEEENENNNNE

9 East and 8 North in any order!

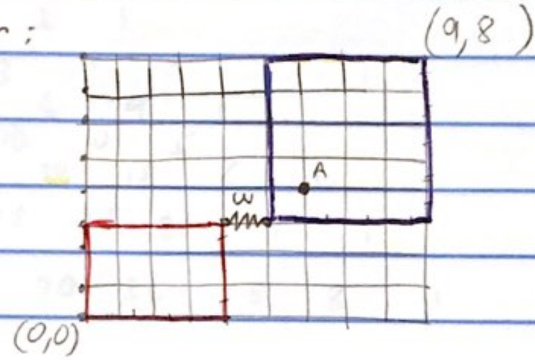
So choose which of the 17 blocks will be East:

Answer $\boxed{\binom{17}{9}} = \boxed{24310}$

... or $\binom{17}{8} = \frac{17!}{8!\,9!}$

2) How many routes are there if the block "w" is "underwater" and cannot be used? (The block starts at (4,3) and goes to (5,3)).

Answer:



→ illegal routes use w: so they go from (0,0) to (4,3) and then use w, then go from (5,3) to (9,8).

$$\binom{17}{9} - \binom{7}{4}\binom{1}{1}\binom{9}{4} = 24310 - 35(126)$$
$$= \boxed{19900}$$

3) How many routes are there if w cannot be used, but we must go through point A = (6,4)?

→ Start with all routes through A. Subtract routes through A and using w.

(0,0) → A → (9,8)   (0,0) → (4,3) → (5,3) → A → (9,8)

$$\boxed{\binom{10}{6}\binom{7}{3} - \binom{7}{4}\binom{1}{1}\binom{2}{1}\binom{7}{3}} = 7350 - 2450 = \boxed{4900}$$

Chapter 5

$\boxed{\text{Binomial}}$ & Multinomial theorems.

Pascal's triangle

Row n		Row sums
0	1	1
1	1 1	2
2	1 2 1	4
3	1 3 3 1	8
4	1 4 6 4 1	16
5	1 5 10 10 5 1	32
6	1 6 15 20 15 6 1	64
7	1 7 21 35 35 21 7 1	128
8	1 8 28 56 70 56 28 8 1	256

$$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$

Row 4 $= \binom{4}{0} \binom{4}{1} \binom{4}{2} \binom{4}{3} \binom{4}{4}$ = numbers of subsets of $\{1, \dots, 4\}$ of given size.

Ex: in $(x+y)(x+y)(x+y)(x+y)$ expanded, how many ways can we make the term x^3y?

Ans: Choose any three of the four $(x+y)$'s to contribute an x, and choose the other $(x+y)$ to contribute a y. So $\binom{4}{3} = \binom{4}{1} = 4$.

Row n : $\underbrace{\binom{n}{0}}_{=1} \quad \underbrace{\binom{n}{1}}_{=n} \ , \dots \quad \underbrace{\binom{n}{k}}_{=\binom{n-1}{k} + \binom{n-1}{k+1}} \dots \quad \underbrace{\binom{n}{n}}_{=1}$ sum: $= 2^n$ = size of $\mathcal{P}\{1, \dots, n\}$

$$\boxed{(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k}$$

Multinomial Theorem

$$(x_1 + x_2 + \dots + x_k)^n$$

$$= \underset{①}{(x_1 + \dots + x_k)}\underset{②}{(x_1 + \dots + x_k)}\underset{③}{(x_1 + \dots + x_k)}\cdots\underset{ⓝ}{(x_1 + \dots + x_k)}$$

$$= x_1^n + n x_1^{n-1} x_2 + n x_1^{n-1} x_3 + \dots + \binom{n}{j_1}\binom{n-j_1}{j_2}\cdots x_1^{j_1} x_2^{j_2} \cdots x_k^{j_k} + \vdots$$

$$\boxed{= \sum_{j_1 + j_2 + \dots + j_k = n} \binom{n}{j_1 \; j_2 \; \dots \; j_k} x_1^{j_1} x_2^{j_2} \cdots x_k^{j_k}}$$

Ex: Find $\displaystyle\sum_{j_1 + j_2 + j_3 = 7} \frac{7!}{j_1! \, j_2! \, j_3!} 2^{j_1} 3^{j_2}$

Ex: Find coefficient of $x^2 y^5$ in $(x + 2y - 3)^9$

Ex: Find $\displaystyle\sum_{i=0}^{5} \frac{5!}{i!(5-i)!} (-3)^i 4^{5-i}$

Ex: Find $\displaystyle\sum_{j_1 + j_2 + j_3 + j_4 = 3} \frac{3!}{j_1! \, j_2! \, j_3! \, j_4!}$

Ex: Find coefficient of $x^3 y^7$ in $(x + y + z - 1)^5$