Common Counting: $n = 20$, $k = 7$, $j_i \geq 0$

\# ways to purchase 20 donuts of 7 different types. } We count the resulting bags of donuts.

\# multi-subsets (multicombinations) of size 20 from $\{\infty \cdot 1, \infty \cdot 2, \dots, \infty \cdot 7\}$

\# ways to distribute 20 identical donuts to 7 knights.

$$\binom{20+7-1}{7-1} = \sum_{j_1 + \dots + j_7 = 20} 1 = \left(\!\!\binom{7}{20}\!\!\right) = \binom{20+7-1}{20}$$

\# Non-negative integer solutions to $x_1 + \dots + x_7 = 20$.

\# ways to <u>plan</u> the numbers of books on 7 shelves, 20 total books (without choosing which books go where.)

ways to purchase 20 donuts of 7 types and give one donut to each of 20 knights.

$$7^{20} = \sum_{j_1+\cdots+j_7=20} \left(\frac{20!}{j_1! \cdots j_7!} \right)$$

shelves

1
2
⋮
7

# of functions from set of size 20 to set of size 7

ways to distribute 20 books on 7 shelves in unordered piles.

anagrams
multi-(permutations)
of
$\{3\cdot a, 2\cdot b, 2\cdot c, 1\cdot d, 9\cdot e, 3\cdot f\}$

$$\frac{20!}{3!\,2!\,0!\,2!\,1!\,9!\,3!} = \binom{20}{3}\binom{17}{2}\binom{15}{0}\binom{15}{2}\binom{13}{1}\binom{12}{9}\binom{3}{3}$$

multinomial (binomial) coefficients $\rightarrow \binom{20}{3\ 2\ 2\ 1\ 9\ 3}$

ways to choose

$$\left.\begin{array}{l} 3 \text{ books for shelf } 1 \\ 2 \text{ books '' '' } 2 \\ \vdots \\ 3 \text{ '' '' '' } 7 \end{array}\right\} \text{unordered piles.}$$

$$\sum_{j_1+\cdots+j_7=20} 20! = \binom{20+7-1}{7-1} 20!$$

$$= (20+7-1)P_7 \qquad = \frac{(20+7-1)!}{(7-1)!}$$

ways to put 20 books on 7 shelves in order.

Common Counting: $n = 20$, $k = 7$, $j_i \geq 0$

ways to purchase 20 donuts of 7 different types. } We count the resulting bags of donuts.

multi-subsets (multicombinations) of size 20 from $\{\infty \cdot 1, \infty \cdot 2, \ldots, \infty \cdot 7\}$

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Non-negative integer solutions to $x_1 + \cdots + x_7 = 20$.

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$$7^{20} = \sum_{j_1 + \cdots + j_7 = 20} \left(\frac{20!}{j_1! \cdots j_7!}\right)$$

shelves

1
2
⋮
7

# of functions from set of size 20 to set of size 7

ways to distribute 20 books on 7 shelves in unordered piles.

anagrams multi-(permutations) of $\{3 \cdot a, 2 \cdot b, 2 \cdot c, 1 \cdot d, 9 \cdot e, 3 \cdot f\}$

$$\frac{20!}{3!\,2!\,0!\,2!\,1!\,9!\,3!} = \binom{20}{3}\binom{17}{2}\binom{15}{0}\binom{15}{2}\binom{13}{1}\binom{12}{9}\binom{3}{3}$$

multinomial (binomial) coefficients $\rightarrow \binom{20}{3\ 2\ 2\ 1\ 9\ 3}$

ways to choose

3 books for shelf 1
2 books " " 2
⋮
3 " " " 7
} unordered piles.

$$\sum_{j_1 + \cdots + j_7 = 20} 20! = \binom{20+7-1}{7-1} 20!$$

$$= (20+7-1)P_7 = \frac{(20+7-1)!}{(7-1)!}$$

ways to put 20 books on 7 shelves in order.