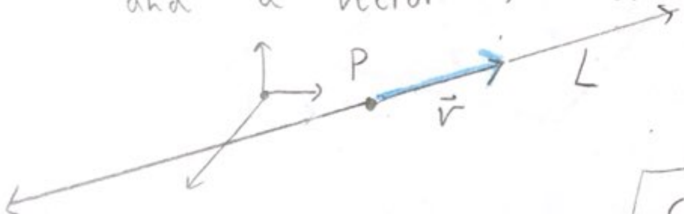
12.4 & 12.5 Lines & Planes using vectors (with dot & cross product.)

Lines in $\mathbb{R}^3$. (uses vector addition)

Need for a line L: a point $P = (x_1, y_1, z_1)$

and a vector for direction $\vec{v} = \langle v_1, v_2, v_3 \rangle$



$L = (x, y, z)$ such that $\boxed{\begin{cases} x = x_1 + v_1 t \\ y = y_1 + v_2 t \\ z = z_1 + v_3 t \end{cases}}$ for $t \in \mathbb{R}$

(these are the parameterized line equations)

Ex: Find L through $P = (1, 4, 0)$
in direction $\vec{v} = \langle 0, -2, 3 \rangle$.

param. $\boxed{L = \begin{cases} x = 1 \\ y = 4 - 2t \\ z = 3t \end{cases}}$

Now remove param: solve each for t, then set solutions for t equal.

$$x = 1 \qquad \frac{y-4}{-2} = t \qquad \frac{z}{3} = t \longrightarrow \boxed{x = 1 \qquad \frac{y-4}{-2} = \frac{z}{3}}$$

only if asked for!

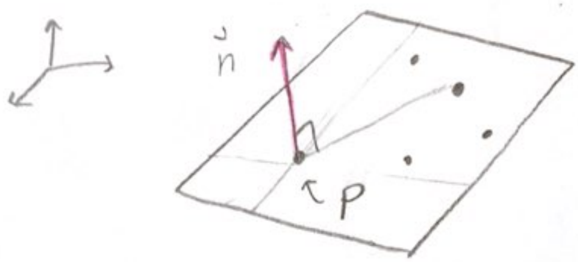
(Get two "=" signs)

[Note: any $\vec{u} \parallel \vec{v}$ and Q any point on L will give the same line: different parameterization, but same equations when parameter is eliminated.]

Line through two points P, Q: use P and $\vec{v} = \overrightarrow{PQ}$.

Planes in $\mathbb{R}^3$. (uses dot)

Need for a plane: a point $P = (x_1, y_1, z_1)$ and a vector $\vec{n} = \langle n_1, n_2, n_3 \rangle$ perpendicular to the plane.

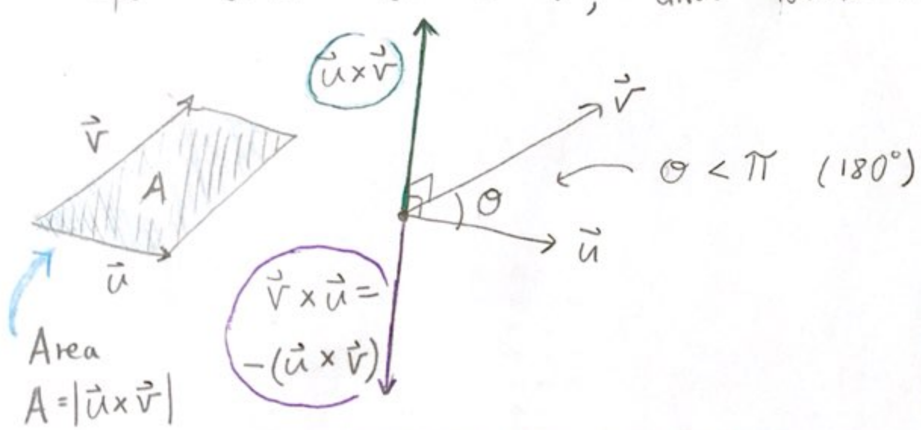


[Any point in the plane and any vector parallel to $\vec{n}$ give the same equation]

Plane $= Q = (x, y, z)$ such that $\vec{n} \cdot \overrightarrow{PQ} = 0$ (orthogonal).

$= (x, y, z)$ such that $\boxed{n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) = 0}$

Recall cross product $\vec{u} \times \vec{v}$. It will be $\perp$ to both $\vec{u}$ & $\vec{v}$, and looks like this:



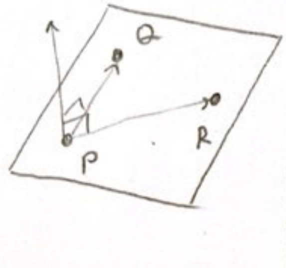
Right-hand rule: for $\theta < \pi$, starting from $\vec{u}$ towards $\vec{v}$ through θ, $\vec{u} \times \vec{v}$ points to me if I see θ as counter clockwise!

Ex: Find plane through 3 points.

$P = (1, 2, 3)$

$Q = (2, 2, 4)$

$R = (1, 3, 3)$



$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$, use P.

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = \langle -1, 0, 1 \rangle$$

Plane: $-1(x-1) + 0(y-2) + 1(z-3) = 0$

$\Rightarrow \quad 1 - x + 0 + z - 3 = 0$

$\Rightarrow \quad \boxed{-x + z = 2}$ check: P, Q, R ✓ ✓ ✓