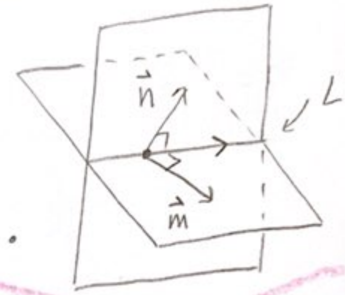
Ex: Find the line of intersection of the two planes;

$3x + y = 4$ and $x - y + 2z = 1$.



method ①

Direction vector $\vec{u}$ of line is in both planes, so $\perp$ to both $\vec{n} = \langle 3, 1, 0 \rangle$ and $\vec{m} = \langle 1, -1, 2 \rangle$.

So $\vec{u} = \vec{n} \times \vec{m} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & 0 \\ 1 & -1 & 2 \end{vmatrix} = \langle 2, -6, -4 \rangle$.

OR $\langle 1, -3, -2 \rangle$

Find a point P in both planes, by trial & error. Let $x = 0$. Then $y = 4$ (first plane)

Then $0 - 4 + 2z = 1$ so $z = \frac{5}{2}$. (second plane)

two equations

Line $\boxed{L = \begin{cases} x = 2t \\ y = 4 - 6t \\ z = \frac{5}{2} - 4t \end{cases}}$, or $\boxed{\frac{x}{2} = \frac{y-4}{-6} = \frac{z - 5/2}{-4}}$

$\Rightarrow \boxed{x = \frac{y-4}{-3} = \frac{z - 5/2}{-2}}$

method ②

Instead: just combine equations to get all variables in terms of x

$$\begin{array}{rl} & 3x + y \qquad = 4 \\ + & x - y + 2z = 1 \\ \hline & 4x \qquad + 2z = 5 \end{array}$$

$y = 4 - 3x$

$\Rightarrow \quad z = \frac{5 - 4x}{2}$

$z = \frac{5}{2} - 2x$

$x = \frac{y-4}{-3}$

$x = \frac{z - 5/2}{-2}$

$\boxed{x = t}$

$L = \begin{cases} x = t \\ y = 4 - 3t \\ z = \frac{5}{2} - 2t \end{cases}$

$\boxed{x = \frac{y-4}{-3} = \frac{z - 5/2}{-2}}$

same!