Calculus III. Review for Test 3, Spring '17.

Also study all the homework and quizzes, as well as examples in class notes.

NOTE: Some questions on the actual test may state "Set up the integral only." Since you don't know which kind, for practice do both the set-up and the integration.

1. Find the integral $\int_1^2 \int_{-2\sqrt{\pi}}^0 \int_0^{\sqrt{4\pi - x^2}} \left(2 + 3\sin(x^2 + y^2)\right) dydxdz.$

   First show the set up of the integral, using only polar variables.

   A:
$$\int_1^2 \int_{\pi/2}^{\pi} \int_0^{2\sqrt{\pi}} (2 + 3\sin(r^2))rdrd\theta dz = 2\pi^2$$

   For the following two problems let $\vec{\mathbf{F}}(x, y, z) = \left\langle 2z,\ x^2y,\ y + x\ln z\right\rangle$.

2. __________ Find $\nabla \cdot \vec{\mathbf{F}}$ at $(1, 0, 2)$. Source or sink?
   The function $\nabla \cdot \vec{\mathbf{F}}(x, y, z) =$__________________________________ .

   A: $\nabla \cdot \vec{\mathbf{F}} = x^2 + x/z,\qquad \nabla \cdot \vec{\mathbf{F}}(1, 0, 2) = 3/2$, which makes it a source.

3. __________ Find $\nabla \times \vec{\mathbf{F}}$ at $(1, 0, 1)$.
   The field $\nabla \times \vec{\mathbf{F}}(x, y, z) =$ __________________________________.

   A: $\nabla \times \vec{\mathbf{F}}(x, y, z) = \langle 1, 2 - \ln z, 2xy\rangle,\qquad \nabla \times \vec{\mathbf{F}}(1, 0, 1) = \langle 1, 2, 0\rangle$.

4. Find $\int_c \nabla f \cdot d\vec{\mathbf{r}}$, where $f(x, y, z) = z + \sin(\ln(xy))$ and C be the line from $(\frac{1}{3}, 3, 7)$ to $(2, \frac{1}{2}, 11)$.

   A: 4

5. Find the volume inside $\rho = 3$ from $\phi = 0$ to $\frac{\pi}{2}$ and $\theta = 0$ to $\frac{\pi}{4}$.

A: $9\pi/4$

6. Let $\vec{\mathbf{F}} = \langle 1,\ ze^y,\ e^y \rangle$ and C be the curve given by $\vec{\mathbf{r}}(t) = \left\langle \sin(\frac{\pi t}{2}),\ t^3 + t^2 + t,\ e^{3t} \right\rangle; 0 \le t \le 1$.
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find f such that $\vec{\mathbf{F}} = \nabla f$.
(c) Find$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

A: (a) $\langle 0, 0, 0 \rangle$ (b) $x + ze^y$ (c) e^6

7. Let $\vec{\mathbf{F}} = \left\langle y^2 + 5 + \sin^2(e^x),\ 2x + \sin^2 y,\ 0 \right\rangle$ and C be the triangular path in the xy-plane from (0,0) to (1,0) to (1,2) and back to (0,0).
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

A: (a) $\langle 0, 0, 2 - 2y \rangle$ (b) $\int_0^1 \int_0^{2x} (2 - 2y) dy dx = 2/3$

8. Let $\vec{\mathbf{F}} = \left\langle e^{3x} - 3y,\ 5y^3 + 1,\ 0 \right\rangle$ and C be the counter-clockwise circle $x^2 + y^2 = 9$.
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

A: (a) $\langle 0, 0, 3 \rangle$ (b) 27π

9. Let $\vec{\mathbf{F}} = \langle e^z,\ 0,\ x + y \rangle$ and C be the line segment from (1,2,1) to (3,1,2).
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

A: (a) $\langle 1, e^z - 1, 0 \rangle$ (b) $2e^2 - 2e + 3.5$

10. Integrate the function $f(x, y, z) = 2x$ over the tetrahedron with vertices $(0, 0, 0)$, $(0, 1, 0)$, $(0, 1, 3)$ and $(4, 1, 0)$.

A:

$$\int_0^4 \int_{x/4}^1 \int_0^{-3x/4+3y} 2x dz dy dx = 4$$

11. Let $\vec{\mathbf{F}} = \left\langle -y^2,\ x,\ z^2 \right\rangle$ and C be the counter-clockwise curve that lies on the surface $g(x, y) = 2 - y$ above the unit circle centered at the origin.
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

A: (a) $\langle 0,\ 0,\ 1 + 2y \rangle$ (b) π

12. Let $\vec{\mathbf{F}} = \left\langle x + y^2,\ y + z^2,\ z + x^2 \right\rangle$ and C be the triangle that goes from (1,0,0) to (0,1,0) to (0,0,1) and back to (1,0,0).
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

A: (a) $\langle -2z, -2x, -2y \rangle$ (b) -1

13. Let $\vec{\mathbf{F}} = \left\langle ze^{xy},\ yz^2,\ x^3 \right\rangle$ and C be the curve on the surface $g(x, y) = 3 + y + 2^x$, where C is above the triangle in the plane that goes from (1,0) to (2,2) to (0,0) and back to (1,0).

Set up $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ using Stoke's theorem and only variables x and y.

A:

$$\int_0^2 \int_y^{\frac{y}{2}+1} \left\langle -2y(3 + y + 2^x),\ e^{xy} - 3x^2, -x(3 + y + 2^x)e^{xy} \right\rangle \cdot \langle -2^x \ln 2,\ -1,\ 1 \rangle \, dxdy$$

$$= \int_0^2 \int_y^{(y/2+1)} (-2y(3 + y + 2^x)(-2^x \ln 2) - (e^{(xy)} - 3x^2) - x(3 + y + 2^x)e^{(xy)}) dxdy$$

14. Find the integral $\displaystyle\int_0^1 \int_{2y}^2 2e^x dxdy$.

A:

$$e^2 + 1$$

15. Find the integral $\displaystyle\int_0^1 \int_{2x}^2 e^{y^2} dydx$.

A:

$$\frac{e^4 - 1}{4}$$