Calculus III. Review for Test 3, Spring '17.

Also study all the homework and quizzes, as well as examples in class notes.

NOTE: Some questions on the actual test may state "Set up the integral only." Since you don't know which kind, for practice do both the set-up and the integration.

1. Find the integral $\int_1^2 \int_{-2\sqrt{\pi}}^{0} \int_0^{\sqrt{4\pi - x^2}} \left(2 + 3\sin(x^2 + y^2)\right) dydxdz.$

   First show the set up of the integral, using only polar variables.

For the following two problems let $\vec{\mathbf{F}}(x, y, z) = \left\langle 2z,\ x^2y,\ y + x\ln z \right\rangle.$

2. __________ Find $\nabla \cdot \vec{\mathbf{F}}$ at $(1, 0, 2)$. Source or sink?
   The function $\nabla \cdot \vec{\mathbf{F}}(x, y, z) =$____________________________________ .

3. __________ Find $\nabla \times \vec{\mathbf{F}}$ at $(1, 0, 1)$.
   The field $\nabla \times \vec{\mathbf{F}}(x, y, z) =$ ____________________________________.

4. Find $\int_c \nabla f \cdot d\vec{\mathbf{r}}$, where $f(x, y, z) = z + \sin(\ln(xy))$ and C be the line from $(\frac{1}{3}, 3, 7)$ to $(2, \frac{1}{2}, 11)$.

5. Find the volume inside $\rho = 3$ from $\phi = 0$ to $\frac{\pi}{2}$ and $\theta = 0$ to $\frac{\pi}{4}$.

6. Let $\vec{\mathbf{F}} = \langle 1,\ ze^y,\ e^y \rangle$ and C be the curve given by $\vec{\mathbf{r}}(t) = \left\langle \sin(\frac{\pi t}{2}),\ t^3 + t^2 + t,\ e^{3t} \right\rangle; 0 \le t \le 1.$
   (a) Find the curl of $\vec{\mathbf{F}}$.
   (b) Find f such that $\vec{\mathbf{F}} = \nabla f$.
   (c) Find$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

7. Let $\vec{\mathbf{F}} = \left\langle y^2 + 5 + \sin^2(e^x),\ 2x + \sin^2 y,\ 0 \right\rangle$ and C be the triangular path in the xy-plane from (0,0) to (1,0) to (1,2) and back to (0,0).
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

8. Let $\vec{\mathbf{F}} = \left\langle e^{3x} - 3y,\ 5y^3 + 1,\ 0 \right\rangle$ and C be the counter-clockwise circle $x^2 + y^2 = 9$.
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

9. Let $\vec{\mathbf{F}} = \langle e^z,\ 0,\ x + y \rangle$ and C be the line segment from (1,2,1) to (3,1,2).
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

10. Integrate the function $f(x, y, z) = 2x$ over the tetrahedron with vertices $(0, 0, 0)$, $(0, 1, 0)$, $(0, 1, 3)$ and $(4, 1, 0)$.

11. Let $\vec{\mathbf{F}} = \left\langle -y^2,\ x,\ z^2 \right\rangle$ and C be the counter-clockwise curve that lies on the surface $g(x, y) = 2 - y$ above the unit circle centered at the origin.
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

12. Let $\vec{\mathbf{F}} = \left\langle x + y^2,\ y + z^2,\ z + x^2 \right\rangle$ and C be the triangle that goes from (1,0,0) to (0,1,0) to (0,0,1) and back to (1,0,0).
(a) Find the curl of $\vec{\mathbf{F}}$.
(b) Find $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

13. Let $\vec{\mathbf{F}} = \left\langle ze^{xy},\ yz^2,\ x^3 \right\rangle$ and C be the curve on the surface $g(x, y) = 3 + y + 2^x$, where C is above the triangle in the plane that goes from (1,0) to (2,2) to (0,0) and back to (1,0).

Set up $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ using Stoke's theorem and only variables x and y.

14. Find the integral $\displaystyle\int_0^1 \int_{2y}^2 2e^x \, dx dy$.

15. Find the integral $\int_0^1 \int_{2x}^2 e^{y^2} dydx$.