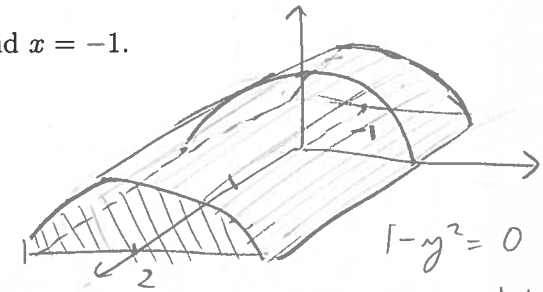
Also study all the homework and quizzes, as well as examples in class notes.

NOTE: Some questions on the actual test may state "Set up the integral only." Since you don't know which kind, for practice do both the set-up and the integration.

1. Find the volume bounded by $z = 1 - y^2$ and the planes $z = 0, x = 2$ and $x = -1$.

   The initial set up of the double integral, using $dA = dydx$ is:



$$\int_{-1}^{2}\int_{-1}^{1} 1-y^2 \, dy\,dx$$

$1-y^2=0$

$y = \pm 1$

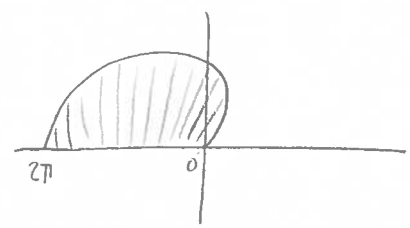
$$= \int_{-1}^{2}\left[y - \frac{y^3}{3}\right]_{-1}^{1} dx \qquad = \int_{-1}^{2} \frac{4}{3}\, dx$$

$$= \int_{-1}^{2} 1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right) dx \qquad = \left[\frac{4}{3}x\right]_{-1}^{2} = \frac{8}{3} + \frac{4}{3}$$

$$= \boxed{4}$$

2. Integrate the function $z = \sqrt{r}$ over the region inside of $r = 2\theta$, where $0 \le \theta \le \pi$.

   The initial set up of the integral, using $dA = rdrd\theta$ is:



$$\int_{0}^{\pi}\int_{0}^{2\theta} \sqrt{r}\; r\, dr\, d\theta$$

$$\int_{0}^{\pi}\int_{0}^{2\theta} r^{3/2}\, dr\, d\theta$$

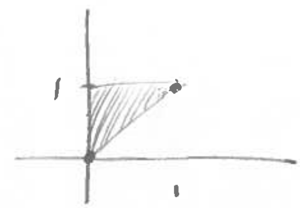
$$= \int_{0}^{\pi}\left[\frac{2r^{5/2}}{5}\right]_{0}^{2\theta} d\theta$$

$$= \int_{0}^{\pi} \frac{2(2^{5/2})\theta^{5/2}}{5} - 0 \; d\theta$$

$$= \left[\frac{2^{7/2}}{5}\left(\frac{2}{7}\theta^{7/2}\right)\right]_{0}^{\pi} = \boxed{\frac{2^{9/2}}{35}\pi^{7/2}}$$

3. Integrate the function $z = y^2 e^{2xy}$ over the triangle with vertices $(0,0)$, $(0,1)$, and $(1,1)$.

The initial set up of the integral, using $dA = dx\,dy$ is:



$$\int_0^1 \int_0^y y^2 e^{2xy}\, dx\, dy$$

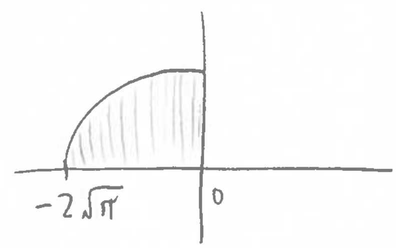
$$= \int_0^1 \left[y^2 \frac{e^{2xy}}{2y} \right]_{x=0}^{y} dy$$

$$= \int_0^1 \left[\frac{y e^{2y^2}}{2} - \frac{y}{2} \right] dy \qquad \left[\begin{array}{l} u = 2y^2 \\ du = 4y\,dy \end{array} \right.$$

$$= \left[\frac{1}{8} e^{2y^2} - \frac{y^2}{4} \right]_0^1 = \frac{1}{8} e^2 - \frac{1}{4} - \frac{1}{8}$$

4. Find the integral $\displaystyle\int_{-2\sqrt{\pi}}^{0} \int_0^{\sqrt{4\pi - x^2}} (2 + 3\sin(x^2 + y^2))\, dy\, dx.$

First show the set up of the integral, using only polar variables and $dA = r\,dr\,d\theta$:



$$\int_{\pi/2}^{\pi} \int_0^{2\sqrt{\pi}} (2 + 3\sin(r^2))\, r\, dr\, d\theta$$

$$= \int_{\pi/2}^{\pi} \int_0^{2\sqrt{\pi}} 2r\, dr\, d\theta + \int_{\pi/2}^{\pi} \int_0^{2\sqrt{\pi}} 3r\sin(r^2)\, dr\, d\theta \qquad \left[\begin{array}{l} u = r^2 \\ du = 2r\,dr \end{array} \right.$$

$$= \int_{\pi/2}^{\pi} \left[r^2 \right]_0^{2\sqrt{\pi}} d\theta + \int_{\pi/2}^{\pi} \left[-\frac{3}{2}\cos(r^2) \right]_0^{2\sqrt{\pi}} d\theta$$

$$= \int_{\pi/2}^{\pi} 4\pi\, d\theta + \int_{\pi/2}^{\pi} -\frac{3}{2}(1 - 1)\, d\theta$$

$$= \left[4\pi\theta \right]_{\pi/2}^{\pi} = 4\pi\left(\frac{\pi}{2}\right) = \boxed{2\pi^2}$$

5. Find the integral $\int_0^2 \int_y^2 (6x+1)dxdy$.

$$= \int_0^2 \left[3x^2 + x\right]_y^2 dy$$

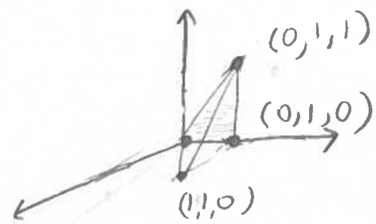
$$= \int_0^2 (12 + 2 - 3y^2 - y)dy$$

$$= \left[14y - y^3 - \frac{y^2}{2}\right]_0^2$$

$$= 28 - 8 - 2 - 0$$

$$= \boxed{18}$$

6. Integrate the function $f(x,y,z) = 2x$ over the tetrahedron with vertices $(0,0,0)$, $(0,1,0)$, $(0,1,1)$ and $(1,1,0)$.



The initial set up of the integral, using $dV = dxdzdy$ is:

$$\int_0^1 \int_0^y \int_0^{y-z} 2x \, dx\,dz\,dy$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \langle -1, 1, -1 \rangle$$

$$-(x-0) + (y-1) - (z-1) = 0$$

$$-x + y - 1 - z + 1 = 0$$

$$x = y - z$$

$$= \int_0^1 \int_0^y \left[x^2\right]_0^{y-z} dz\,dy$$

$$= \int_0^1 \int_0^y y^2 - 2yz + z^2 \, dz\,dy$$

$$= \int_0^1 \left[zy^2 - yz^2 + \frac{z^3}{3}\right]_0^y dy$$

$$= \int_0^1 y^3 - y^3 + \frac{y^3}{3} \, dy = \left[\frac{y^4}{12}\right]_0^1 = \boxed{\frac{1}{12}}$$

7. Find the volume integral $\int_0^{\pi/6} \int_0^{\pi/2} \int_0^3 \rho^2 \sin\phi \, d\rho d\theta d\phi$.

$$= \int_0^{\pi/6} \int_0^{\pi/2} \left[\frac{\rho^3}{3} \sin\phi \, d\theta \, d\phi\right]_0^3$$

$$= \int_0^{\pi/6} \int_0^{\pi/2} 9 \sin\phi \, d\theta \, d\phi$$

$$= \int_0^{\pi/6} \left[9\theta \sin\phi\right]_0^{\pi/2} d\phi$$

$$= \int_0^{\pi/6} \frac{9\pi}{2} \sin\phi \, d\phi$$

$$= \left[\frac{9\pi}{2}(-\cos\phi)\right]_0^{\pi/6}$$

$$= \frac{9\pi}{2}\left(-\frac{\sqrt{3}}{2} - (-1)\right)$$

$$= \boxed{\frac{9\pi}{2}\left(1 - \frac{\sqrt{3}}{2}\right)}$$