Calculus III. Test 2, Additional Review

1. For each function $z = f(x, y)$ find two partial derivatives: f_x and f_y.

(a) $f(x, y) = \dfrac{\ln y}{x}$

$f_x = \boxed{\dfrac{-\ln y}{x^2}}$ $f_y = \boxed{\dfrac{1}{xy}}$

(b) $f(x, y) = (y - 2)^{(e^x)}$

$f_x = \boxed{(y-2)^{e^x} \ln(y-2)\, e^x}$ $f_y = \boxed{e^x (y-2)^{(e^x - 1)}}$

(c) $f(x, y) = \sqrt[2y]{x - 2} = (x-2)^{\left(\frac{1}{2y}\right)}$

$f_x = \boxed{\dfrac{1}{2y}(x-2)^{\left(\frac{1}{2y} - 1\right)}}$ $f_y = \boxed{(x-2)^{\frac{1}{2y}} \ln(x-2)\left(\dfrac{-1}{2y^2}\right)}$

(d) $f(x, y) = \sin(xy)$

$f_x = \boxed{y(\cos(xy))}$ $f_y = \boxed{x(\cos(xy))}$

(e) $f(x, y) = x^2 6^y$

$f_x = \boxed{2x6^y}$ $f_y = \boxed{x^2 6^y \ln 6}$

2. **Use Lagrange Multipliers to find the absolute min and max of** $f(x,y) = 5^{(2x+3y)}$ **on the curve** $x^2 + 3y^2 = 28$.

Use $f_x = 5^{(2x+3y)} 2\ln 5$; $\quad f_y = 5^{(2x+3y)} 3\ln 5$

1

$5^{2x+3y} 2\ln 5 = c\,2x$

$5^{2x+3y} \ln 5 = cx$

2

$5^{2x+3y} 3\ln 5 = c\,6y$

$5^{2x+3y} \ln 5 = c2y$

$cx = c2y$

$c = 0$ leads to $5^{2x+3y} = 0$ in 1 $\times$

$x = 2y$

use

3

$x^2 + 3y^2 = 28$

$(2y)^2 + 3y^2 = 28$

$4y^2 + 3y^2 = 28$

$7y^2 = 28$

$y^2 = 4$

$y = 2$	$y = -2$
$x = 4$	$x = -4$

c.p.	z
$(4, 2)$	$5^{8+6} = \boxed{5^{14} \quad \text{max}}$
$(-4, -2)$	$5^{-8-6} = \boxed{5^{-14} \quad \text{min}}$

3. Given $z = f(x,y)$ has a critical point where both partials are 0 at $(1,2)$.

Also $f(7,8) = 15$ $\quad$ $f_{xx}(1,2) = 3$

and $f_x(7,8) = 5$ $\quad$ $f_{yy}(1,2) = 4$

and $f_y(7,8) = -2$ $\quad$ $f_{xy}(1,2) = 5$

a) Find the directional derivative of f over $(7,8)$ in the direction of $\vec{v} = \langle 3, -1 \rangle$.

$$D_{\vec{v}} f = \frac{\nabla f \cdot \vec{v}}{|\vec{v}|} = \frac{\langle 5,-2\rangle \cdot \langle 3,-1\rangle}{\sqrt{9+1}} = \boxed{\frac{17}{\sqrt{10}}}$$

b) Find the 2d direction vector of max decrease for $f(x,y)$ over $(x,y) = (7,8)$.

$$-\nabla f = \boxed{\langle -5, 2\rangle}$$

c) Find the max rate of increase for f over $(7,8)$.

$$|\nabla f| = |\langle 5,-2\rangle| = \sqrt{25+4} = \boxed{\sqrt{29}}$$

d) Find whether the point on f over $(1,2)$ is a local max, local min, saddle or inconclusive. Show the value of D.

$$D = f_{xx} f_{yy} - (f_{xy})^2 = 3(4) - 5^2 = 12 - 25 = \boxed{-13}$$

$\boxed{\text{saddle}}$

e) For a curve $\vec{r}(t)$ obeying $\vec{r}(5) = \langle 7, 8 \rangle$ and $\vec{r}'(5) = \langle 3, -7 \rangle$; find $\frac{dz}{dt}$ when $t = 5$.

$$\frac{dz}{dt} = \nabla f \cdot \vec{r}' = \langle 5,-2\rangle \cdot \langle 3,-7\rangle = \boxed{29}$$

4. **Given $z = f(x,y)$ has a horizontal tangent plane over $(1,2)$ and it has tangent plane given by $3x - 2y - z = 5$ over $(7,8)$.**

Also $f_{xx}(1,2) = -2$;
and $f_{yy}(1,2) = -3$;
and $f_{xy}(1,2) = 1$

$z = 3x - 2y - 5 \Rightarrow z_0 = 3(7) - 2(8) - 5 = 0$

a) Find the instantaneous rate of change in z at $(7,8)$, as x increases and y is held constant at 8.

$f_x(7,8) = \boxed{3}$

since $\vec{n} = \langle \underset{f_x}{3}, \underset{f_y}{-2}, -1 \rangle$

b) Find the instantaneous rate of change in z with respect to t at $t = 0$ where (x,y) is on the curve $\vec{\mathbf{r}}(t) = \langle t^2 + 7, 8e^t, \rangle$.

$\vec{r}\,' = \langle 2t, 8e^t \rangle$

$\vec{r}\,'(0) = \langle 0, 8 \rangle$

$\frac{dz}{dt} = \nabla f \cdot \vec{r}\,'$

$= \langle 3, -2 \rangle \cdot \langle 0, 8 \rangle = \boxed{-16}$

c) Use the linearization of $f(x,y)$ over $(7,8)$ to approximate $f(7.1, 7.9)$.

$L(7.1, 7.9) = f_x(7.1 - 7) + f_y(7.9 - 8) + z_0$

$= 3(0.1) + -2(-0.1) + 0 = \boxed{0.5}$

$\boxed{\text{OR} \quad z = 3(7.1) - 2(7.9) - 5 = 0.5}$

d) Find whether the point on f over $(1,2)$ is a local max, local min, saddle or inconclusive. Show the value of D.

$D = -2(-3) - (1)^2 = \boxed{5} > 0$

and $f_{xx} = -2 < 0$, so $\boxed{\text{max}}$

e) Find the normal vector to the tangent plane of f at $(1,2)$. horizontal (given)

so ... $\boxed{\vec{n} = \langle 0, 0, -1 \rangle}$