Calculus III. Test 2, Additional Review

1. For each function $z = f(x, y)$ find two partial derivatives: f_x and f_y.

(a) $f(x, y) = \dfrac{\ln y}{x}$

$f_x =$ $f_y =$

(b) $f(x, y) = (y - 2)^{(e^x)}$

$f_x =$ $f_y =$

(c) $f(x, y) = \sqrt[2y]{x - 2}$

$f_x =$ $f_y =$

(d) $f(x, y) = \sin(xy)$

$f_x =$ $f_y =$

(e) $f(x, y) = x^2 6^y$

$f_x =$ $f_y =$

2. Use Lagrange Multipliers to find the absolute min and max of $f(x,y) = 5^{(2x+3y)}$ **on the curve** $x^2 + 3y^2 = 28$.

Use $f_x = 5^{(2x+3y)} 2 \ln 5$; $\quad f_y = 5^{(2x+3y)} 3 \ln 5$

3. Given $z = f(x, y)$ has a critical point where both partials are 0 at $(1, 2)$.

Also $f(7, 8) = 15$ $\quad$ $f_{xx}(1, 2) = 3$

and $f_x(7, 8) = 5$ $\quad$ $f_{yy}(1, 2) = 4$

and $f_y(7, 8) = -2$ $\quad$ $f_{xy}(1, 2) = 5$

a) Find the directional derivative of f over $(7, 8)$ in the direction of $\vec{\mathbf{v}} = \langle 3, -1 \rangle$.

b) Find the 2d direction vector of max decrease for $f(x, y)$ over $(x, y) = (7, 8)$.

c) Find the max rate of increase for f over $(7, 8)$.

d) Find whether the point on f over $(1, 2)$ is a local max, local min, saddle or inconclusive. Show the value of D.

e) For a curve $\vec{\mathbf{r}}(t)$ obeying $\vec{\mathbf{r}}(5) = \langle 7, 8 \rangle$ and $\vec{\mathbf{r}'}(5) = \langle 3, -7 \rangle$; find $\frac{dz}{dt}$ when $t = 5$.

4. Given $z = f(x, y)$ has a horizontal tangent plane over $(1, 2)$ and it has tangent plane given by $3x - 2y - z = 5$ over $(7, 8)$.

Also $f_{xx}(1, 2) = -2$;
and $f_{yy}(1, 2) = -3$;
and $f_{xy}(1, 2) = 1$

a) Find the instantaneous rate of change in z at $(7, 8)$, as x increases and y is held constant at 8.

b) Find the instantaneous rate of change in z with respect to t at $t = 0$ where (x, y) is on the curve $\vec{\mathbf{r}}(t) = \langle t^2 + 7, 8e^t, \rangle$.

c) Use the linearization of $f(x, y)$ over $(7, 8)$ to approximate $f(7.1, 7.9)$.

d) Find whether the point on f over $(1, 2)$ is a local max, local min, saddle or inconclusive. Show the value of D.

e) Find the normal vector to the tangent plane of f at $(1, 2)$.